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界面角之破壞分析(2/3)
Failure Analysis of Interface Corners (2/3)

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中文摘要

前一年計畫中我們成功地定義出一同時適用於均質裂縫，界面裂縫，均質角與界面角之應力強度因子。本年度則更進一步地提出一穩定可靠之計算應力強度因子的方法。以 Betti and Rayleigh 交互理論為基礎，本計畫建立一與路徑無關之 H 積分，亦提供該積分所需之輔助解。為驗證此一計算方法之穩定性及效率，本年度我們提前進行了許多例證如: 於均質等向性或異向性材料中之裂縫、楔形角、界面裂縫及界面角等。

關鍵詞：界面角、應力強度因子、破壞分析、異向性彈性力學

Abstract

In the first year’s study, a unified definition for the stress intensity factors was proposed for the general interface corners. Through this unified definition, a direct connection between homogenous cracks, interface cracks, homogeneous corners ad interface corners has been built. To provide a stable and efficient computing approach for the general mixed-mode stress intensity factors, the path-independent $H$-integral based on reciprocal theorem of Betti and Rayleigh is established in this year’s project. The complementary solutions needed for calculation of $H$-integral are provided in this study. To illustrate our results, several different kinds of examples are shown such as cracks in homogenous isotropic or anisotropic materials, central or edge notches in isotropic materials, interface cracks and interface corners between two dissimilar materials.

Keywords: interface corner, stress intensity factor, failure analysis, anisotropic elasticity

1. Introduction

In our first year study, a proper definition for the stress intensity factors, $K_I, K_{II}, K_{III}$, associated with the most critical singular order $(\delta_R + i \varepsilon_R)$ of the interface corners has been proposed as

$$
\begin{align}
\begin{bmatrix} K_{II} \\ K_I \\ K_{III} \end{bmatrix} &= \lim_{r \to 0} \sqrt{2 \pi r} \delta_{ij} \Lambda < (r / \ell)^{-i \varepsilon_R} > \Lambda^{-1} \begin{bmatrix} \sigma_{r \rho} \\ \sigma_{\theta \theta} \\ \sigma_{3 \theta} \end{bmatrix}, \\
& \text{or in matrix form} \\
k &= \lim_{r \to 0} \sqrt{2 \pi r} \delta_{ij} \Lambda < (r / \ell)^{-i \varepsilon_R} > \Lambda^{-1} \phi, (r,0). 
\end{align}
$$

In this definition, $(r, \theta)$ is the polar coordinate with origin located on the corner tip; $\sigma_{ij}$ and $\phi$ are stresses and stress function vector, $\Lambda$ is a matrix related to the wedge configurations and properties, whose details are given in the report of our first year project; $\ell$ is a reference length used to smooth the physical unit of the stress intensity factors associated with the complex singular orders; the superscript -1 denotes the inverse of a matrix; the subscript comma denotes differentiation and the angular bracket $<>$ stands for a diagonal matrix in which each component is varied according to the subscript $\alpha$, e.g., $< z_{\alpha} > = \text{diag.}[z_1, z_2, z_3]$.

2. Path-independent H-integral

According to the definition of the stress intensity factors proposed in (1), to calculate their values we need to know the stresses near the tip of interface corners. However, due to the singular and possibly oscillatory behaviors of the near tip solutions for interface corners, it is not easy to get convergent values for the stress intensity factors directly from the definition (1). To overcome this problem, several path-independent integrals have been proposed for the special cases of interface corners such as $J$-integral (Rice, 1968), $L$-Integral (Choi and Earmme, 1992), $M$-integral (Im and Kim, 2000) and $H$-integral (Sinclair, et al., 1984) for crack problems. Since these
integrals have a special feature that they are independent of paths, the complexity of stresses around the crack tip can then be avoided. The interface corners are usually in the status of mixed-mode intensity. Thus, employing $H$-integral to compute the stress intensity factors defined in (1) may be a good choice.

The path-independent $H$-integral is based on the reciprocal theorem of Betti and Rayleigh (Sokolnikoff, 1956). It states that: if an elastic body is subjected to two systems of body and surface forces, then the work that would be done by the first system in acting through the displacements due to the second system of forces is equal to the work that would be done by the second system in acting through the displacements due to the first system of forces. If we choose the first system to be the (actual) one we consider, and the second system to be the complementary (or called virtual) one. In the absence of body forces, this theorem can be written in the following form

$$\int_C (\mathbf{u}^T \hat{\mathbf{t}} - \hat{\mathbf{u}}^T \mathbf{t}) ds = 0,$$

where $\mathbf{u}$ and $\mathbf{t}$ are the displacement and traction vectors of the actual system, and $\hat{\mathbf{u}}$ and $\hat{\mathbf{t}}$ are those of the complementary system. $C$ is any closed contour in a simply connected region, which is selected to be $C_c + C_1 + C_\infty + C_2$ as shown in Figure 1. Because the two outer surfaces of the multibonded wedges are considered to be free of tractions, $\mathbf{t} = \hat{\mathbf{t}} = \mathbf{0}$ along $C_1$ and $C_2$, and hence,

$$\int_{C_\infty} (\mathbf{u}^T \hat{\mathbf{t}} - \hat{\mathbf{u}}^T \mathbf{t}) ds = -\int_{C_c} (\mathbf{u}^T \hat{\mathbf{t}} - \hat{\mathbf{u}}^T \mathbf{t}) ds = \int_{C_c} (\mathbf{u}^T \hat{\mathbf{t}} - \hat{\mathbf{u}}^T \mathbf{t}) ds,$$

where both $C_c$ and $C_\infty$ are the paths emanate from the lower wedge flank ($\theta = \theta_0$) to the upper wedge flank ($\theta = \theta_n$) counterclockwisely. In other words, the $H$-integral defined by

$$H = \int_{\Gamma} (\mathbf{u}^T \hat{\mathbf{t}} - \hat{\mathbf{u}}^T \mathbf{t}) ds,$$

is path-independent for free-free multibonded wedges when the path $\Gamma$ emanates from $\theta_0$ and terminates on $\theta_n$ in counterclockwise direction.

By shrinking the inner path $C_c$ inside the region dominated by the singular field and making a judicious choice for the complementary solution, we can get an analytical expression for the $H$-integral in terms of the coefficients $c_i$ (or simply $c$) which have a direct relation with the stress intensity factors. Thus, if one can evaluate the $H$-integral from the other path far from the tip, through the path-independent property shown in (3) we can calculate the stress intensity factors. With this understanding, we will now try to find the suitable complementary solutions and then derive formulae for the stress intensity factors.

Since the integral path can be selected arbitrarily from the lower wedge flank $\theta_0$ to upper wedge flank $\theta_n$, for simplicity we choose a circular counterclockwise path through the region dominated by the singular field. Along this path, the traction $\mathbf{t} = \mathbf{\Phi}_\theta / r$ and $ds = rd\theta$, so equation (4) becomes

$$H = \int_{\theta_0}^{\theta_n} (\mathbf{u}^T \hat{\mathbf{\Phi}}_\theta - \hat{\mathbf{u}}^T \mathbf{\Phi}_\theta) d\theta.$$

Substituting the near tip solution and its associated complementary solution into (5), and using the path-independent property proved in (3), we get (Hwu and Kuo, 2007)

$$\mathbf{k} = \sqrt{2\pi} \Lambda < (\lambda_\infty + i\epsilon_\infty)\ell^{mc} > \mathbf{H}^{-1} \mathbf{h},$$

where $\mathbf{h}$ is a vector composed of the values of $H$-integral calculated from (5) with certain coefficients of complementary solutions being assumed to be zero; $\mathbf{H}^*$ is a matrix whose
components $H_{ij}^*$ is defined as

$$H_{ij}^* = \int \left[ \hat{p}_i^T(\theta)\hat{q}_j'(\theta) - \hat{p}_j^T(\theta)\hat{q}_i'(\theta) \right] d\theta, \quad i, j = 1, 2, \ldots \quad (7)$$

In the above, $p_i(\theta)$ and $q_i(\theta)$, $i = 1, 2, 3$ are functions related to the near tip solutions, whereas $\hat{p}_i(\theta)$ and $\hat{q}_i(\theta)$ are functions related to the complementary solutions, whose details can be found in (Hwu and Kuo, 2007).

3. Numerical Examples

A unified definition for the stress intensity factors of cracks, interface cracks and interface corners is proposed in equation (1). No matter what kind of stress singularity occurs for the general interface corners/cracks, (distinct or repeated, real or complex), an efficient and stable approach of calculating the stress intensity factors is suggested by using the relation (6) with the path-independent $H$-integral defined in (4). Detailed studies about the convergency and efficiency of the $H$-integral as well as the validity of the range and shape of the path have been done through several different kinds of examples. To save the space of this report, only selected examples are shown to illustrate the versatility of the unified definition, such as (1) cracks in homogeneous isotropic or anisotropic materials, (2) central or edge notch in isotropic materials, (3) interface cracks between two dissimilar isotropic materials, and (4) interface corners between two dissimilar materials. In the first example, the singular order is $-1/2$ which is a repeated root associated with opening, shearing and tearing stress intensity factors. Example 2 is a case of single wedge problem whose singular order is generally less than that of crack (here, we compare the absolute value of the singular order), and is generally real. Example 3 shows the variation of the stress intensity factors of the interface cracks versus the stiffness ratio of the two materials. No matter what kinds of combination of the bimaterials, the singular orders of this example are always $-1/2$ and $-1/2 \pm i \epsilon$. The last example shows a typical example of interface corners, which occurs frequently in electric devices. The singular order of this case is generally complex.

Note that in the following calculation, the stresses and displacements of the actual system are obtained from the commercial finite element software ANSYS. For convenience, the paths are usually selected to pass through Gauss points or nodal points. Otherwise, interpolation technique is used to get the values of displacements and stresses. In our example all the integration paths are
selected to pass through nodal points. Since the numerical output will depend on element meshes and integral paths, both studies have been done before performing the following examples. Our results show that the convergent and stable values will be obtained if the normalized element size \( b/a \) is less than 0.05 and the normalized integral paths \( r/a \) lie within \( 0.2 < r/a < 0.8 \), where \( b \) is the grid size of the meshes in the region \( 2a \times 2a \) centered on the crack/notch tips, \( r \) is the radius of circular integral path and \( a \) is the crack or notch length.

It should be noted that although the path independency of \( H \)-integral has been proved theoretically in the previous Section, numerical studies through several different shapes and ranges of paths show that the stress intensity factors calculated from the paths with \( r/a \) less than 0.2 become unstable and will change rapidly, which may come from the incorrect stress information near the crack/notch tip provided by FEA. Therefore, when using \( H \)-integral to calculate the stress intensity factors we avoid to take the values near the range of \( r/a < 0.2 \), which is the advantage of the path-independent integrals.

Figures 2-6 are some examples we did in this project and Figure 7 is a typical result for the interface corner problems. One may refer to (Hwu and Kuo, 2007) for detailed description of these examples.

Figure 2. Schematic diagram of an edge crack in an homogeneous isotropic plate subjected to (a) uniform tension; (b) uniform end shear.

Figure 3. Schematic diagram of a central crack in an anisotropic plate
Figure 4. Schematic diagrams of notches in isotropic plates: (a) central notch (b) edge notch.

Figure 5. Interface cracks between two dissimilar isotropic materials: (a) central crack (b) edge crack.
Figure 6. Schematic diagrams of interface corner between dissimilar materials.

Figure 7(a): Orders of stress singularity $\delta$ versus angles of interface corner $\alpha$.

Figure 7(b): Stress intensity factors versus angles of interface corner $\alpha$.

6. Conclusions

The near tip solutions for the general interface corners have been divided into five categories depending on whether the singular order is distinct or repeated, real or complex. These five categories cover all the possibilities of the interface corners including the homogenous cracks and
interface cracks. Based upon the conventional definitions for the cracks in homogeneous materials and the interface cracks in bimaterials, a unified definition for the stress intensity factors of general interface corners and cracks is proposed in (1). With the knowledge of the near tip solutions, an important relation connecting this newly defined stress intensity factor and the path-independent $H$-integral is obtained in (6). To calculate the stress intensity factors through this relation, both the near tip solutions associated with the critical singular order $\lambda_c$ and the complementary solutions associated with $-\lambda_c$ are needed. With these solutions, the $H$-integral can be calculated effectively by inputting the displacements and tractions of the actual state directly from any numerical method such as finite element or boundary element method. To illustrate the versatility of the unified definition of the stress intensity factors and the accuracy and efficiency of the $H$-integral, four different kinds of examples are shown such as cracks/notches in homogeneous materials and interface cracks/corners between dissimilar materials.

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References