Abstract

This project works on the estimate of mechanical properties of four different scale structures which are unsymmetric laminated composites, tapered composite wings, flapping micro-air-vehicles, multi-walled carbon nanotubes. The efforts have been made to establish a unified approach for all multi-scale structures able to be proposed through the experience of this research. Aiming at this goal, in the first year we have completed the following works: 1. the Green’s function of unsymmetric laminates, 2. finite element modeling of tapered composite wings, 3. numerical simulation of flapping wings, 4. equivalent structural analysis of carbon nanotube. This year the follow-up works have also been completed: 1. BEM design for coupled stretching-bending analysis, 2. FEM design for tapered composite wings, 3. lift of flapping wings, 4. Van der Waals force modelling, etc.

Keywords: Unsymmetric Laminated Composites, Tapered Composite Wings, Flapping Micro-Air-Vehicles, Multi-Walled Carbon Nanotubes, Mechanical Properties, On-Line Measurement, Multi-Scale Structures

1. BEM Design for Coupled Stretching-Bending Analysis

Consider the bending-stretching coupling analysis of composite laminates, no associated boundary integral equations, which are the basis for the programming of boundary element codes, can be found in the literature. Only the boundary integral equations for the pure bending or pure stretching analysis have been presented. According to the reciprocal theorem of Betti and Raleigh, the general boundary integral equations for the bending-stretching coupling analysis of composite laminates are derived in this report.

The reciprocal theorem of Betti and Raleigh in terms of stresses and strains can be expressed as:

\[\int_{\Omega} \sigma_{ij} \varepsilon_{ij}^* \, d\Omega = \int_{\Omega} \sigma_{ij}^* \varepsilon_{ij} \, d\Omega,\]

where \(\sigma_{ij}, \varepsilon_{ij}\) and \(\sigma_{ij}^*, \varepsilon_{ij}^*\), \(i, j = 1, 2, 3\), are the stresses and strains induced by two different loading systems on the same elastic body whose region is denoted by \(\Omega\). If the elastic body is a moderately thick laminated plate, according to the Mindlin’s assumption the integration of (1) with respect to the thickness may lead to an expression in terms of stress resultants \((N_x, N_y, N_w)\), bending moments \((M_x, M_y, M_w)\), transverse shear forces \((Q_x, Q_y, Q_w)\), midplane strains \((\varepsilon_{10}, \varepsilon_{20}, \varepsilon_{30})\), curvatures \((\kappa_x, \kappa_y, \kappa_z)\) and transverse shear strains \((\gamma_{12}, \gamma_{13}, \gamma_{23})\). Employing the kinematic relations between midplane displacements and strains/curvatures, the technique of integration by parts, the equilibrium equations of the plates, the transformation equations for vectors and tensors, we can now further lead equation (1) to the following expression (Hwu and Liang, 2006):
\[
\int (q_x u_o^* + q_y v_o^* + q w^* + m_x \beta_x^* + m_y \beta_y^*) dA \\
+ \int (N_x u_o^* + N_y u^* + M_x \beta_x^* + M_y \beta_y^* + Q_x w^*) d\Gamma \\
= \int (q_x u_o^* + q_y v_o^* + q w^* + m_x \beta_x^* + m_y \beta_y^*) dA \\
+ \int (N_x u_o^* + N_y u^* + M_x \beta_x^* + M_y \beta_y^* + Q_x w^*) d\Gamma.
\]

(2)

For thin plates, Kirchhoff's assumptions apply and transverse shear deformations are usually ignored, i.e.,
\[
\gamma_{sz} = \gamma_{sy} = 0.
\]

(3)

With this assumption, the rotation angles will not be independent but be related to the deflection \(w\) by
\[
\beta_x = \frac{\partial w}{\partial n}, \quad \beta_y = -\frac{\partial w}{\partial n}.
\]

(4)

Substituting (4) into (2) and integrating by parts, we get
\[
\int (q_x u_o^* + q_y v_o^* + q w^* + m_x \beta_x^* + m_y \beta_y^*) dA \\
+ \int \left[ (N_x u_o^* + N_y u^* + M_x \beta_x^* + M_y \beta_y^* + V_x w) - (M_x w^* - V_x w^*) \right] d\Gamma \\
= \int (q_x u_o^* + q_y v_o^* + q w^* + m_x \beta_x^* + m_y \beta_y^*) dA \\
+ \int \left[ (N_x u_o^* + N_y u^* + M_x \beta_x^* + M_y \beta_y^* + V_x w) - (M_x w^* - V_x w^*) \right] d\Gamma,
\]

(5)

where \(V_x\) is the effective shear force defined as
\[
\frac{\partial Q_x}{\partial n} + \frac{2M_{xx}}{\partial n},
\]

(6)

and \(\Gamma^-\) and \(\Gamma^+\) represent, respectively, the starting and final points of the boundary \(\Gamma\). If the \(M_{xx} w^*\) or \(M_{xx} w\) value is continuous, the last terms of both sides of (5) will vanish. Otherwise, the addition of these two terms becomes necessary, which may occur if the \(M_{xx}\) value is discontinuous. When the boundary has many corners, the last terms of both sides of (5) may be represented as
\[
(M_{xx} w^*)_{\Gamma^-}^- = \sum_{i=1}^k (M_{xx}^- - M_{xx})_i w_k^*,
\]

(7)

\[
(M_{xx} w^*)_{\Gamma^+}^+ = \sum_{i=1}^k (M_{xx}^+ - M_{xx})_i w_k^*.
\]

where \(i\) is the number of corners. (Figure 1)

Consider the body force \(q^* = (q_x^*, q_y^*, q_w^*, m_n^*)\)
be a unit point load or moment applied at the point \(\xi\). The load is applied in each of three orthogonal directions \(e_i, i = 1, 2, 3\), while the moment is applied in the direction of \(n\), i.e.,
\[
q_i(x) = \delta(\xi, x)e_i,
\]

(8)

where \(\delta(\xi, x)\) represents the Dirac delta function, \(\xi\) is the singular load point and \(x \in A\) is the field point. Substituting each point load of (8) independently into (5) and using (7), we get
\[
\begin{align*}
&\left( u_i(\xi) + \int_{\Gamma} \left[ u_j^*(\xi, x) u_j(x) d\Gamma(x) - \sum_{i=1}^k t_i^*(\xi, x_i) u_i(x_i) \right] \right) \\
&= \int_{\Gamma} u_j^*(\xi, x) u_j(x) d\Gamma(x) + \int_{A} u_j^*(\xi, x) q_j(x) dA(x) \\
&= \sum_{i=1}^k u_j^*(\xi, x_i) t_j(x_i), \quad i, j = 1, 2, 3, 4,
\end{align*}
\]

(9)

in which new notations are used for the convenience of later presentation. They are
\[
\begin{align*}
t_1 &= u_{\xi}, \quad t_2 = v_{\xi}, \quad t_3 = w, \quad t_4 = \beta_n, \\
t_1 &= T_x = N_x n_x + N_y n_y, \quad t_2 = T_y = N_y n_x + N_x n_y, \\
t_3 &= V_n, \quad t_4 = M_n, \\
q_1 &= q_{\xi}, \quad q_2 = q_x, \quad q_3 = q_y, \quad q_4 = m_n,
\end{align*}
\]

(10)

and \(u_j^*(\xi, x), t_j^*(\xi, x)\) and \(t_j^*(\xi, x)\) represent \(u_j, t_j\) and \(t_j\) at point \(x\) corresponding to a unit
point force acting in the \( e_i \) direction applied at point \( \xi \). Since \( t_j' (\xi, x) \) are the stress resultants corresponding to the unit point load applied at \( \xi \) directing in \( e_i \), when \( x \) approaches to \( \xi \) they will become singular. Therefore, if \( \xi \) goes to the boundary \( \Gamma \), the integral on the right hand side of (9) should be taken in the sense of Cauchy principal value and equation (9) can be rewritten as

\[
c_i (\xi) u_i (\xi) + \int r^j (\xi, x) u_j (x) dy - \sum_{k=1}^{N} c_{ik} (\xi) u_k (x) = 0
\]

where \( c_i (\xi) \) is the coefficient containing the corresponding principal value which can be indirectly computed by applying (19) to represent rigid body movements.

Notice that we have now four unknown functions, i.e., \( u_j \) or \( t_j \), \( j = 1,2,3,4 \), and four equations (11). The four equations provided in (11), called the boundary integral equations, then constitute the basis of the boundary element formulation. To make these equations work for the programming of boundary element codes, the fundamental solutions provided in the first year project will then play an important role in BEM design.

2. FEM Design for Tapered Composite Wings

In the first year of this project, by following the standard approach for finite element formulation, we have obtained

\[
\pi - \kappa = \frac{f(t)}{2} u_j' [K_e - J_e] u_j - f(t) u_j f_e
\]

where \( \pi \) is potential energy and \( \kappa \) is the kinetic energy, \( u_j \) denotes the element displacement, \( K_e \), \( J_e \), and \( f_e \) are element stiffness matrix, element inertia matrix, and element external force vector, respectively. Explanation of the detailed derivation and the symbols used above can be found in (Yu and Hwu, 2006) (Figure 2).

Based upon this result, FEM for tapered composite wings has been designed in this year for both divergence and free vibration problems.

**Divergence**

In our project, we use aerodynamic strip theory and the known results for two-dimensional flow to approximate the lift and the pitching moment. With this approximation, the external aerodynamic force vector \( f_e \) can be written in terms of displacement as

\[
f_e = \int N^T (y) p(y) dy = q_n [p_0 + K_e u_e]
\]

where \( q_n \) denotes normal dynamic pressure, \( p_0 \) is a constant vector, and \( K_e \) is the aerodynamic stiffness matrix related to the lift and pitching moment. Substituting (13) into (12), and setting \( f(t) = 1 \) and neglecting \( J_e \) for static problem, we obtain

\[
\pi - \kappa = \frac{1}{2} u_j' [K_e - J_e] u_j - q_n [p_0 + K_e u_e]
\]

The divergence dynamic pressure can then be obtained from (14) by taking derivative of (14) with respect to \( u_e \) and letting the determinant of the coefficient matrix of \( u_e \) vanish. The final result is

\[
\frac{1}{2} [K_e + K_e^T] - q_n [K_e + K_e^T] = 0
\]

**Free Vibration**

To determine the natural frequency of the tapered composite wings, the values of all components related to the external forces can be set to zero, and hence \( f_e = 0 \). Furthermore, \( f(t) \) is assumed as a harmonic motion with the natural frequency \( \omega \) in the free vibration analysis. By using Eq. (12) and Hamilton’s principle again, the equation of motion is derived as

\[
\frac{1}{2} [K_e + K_e^T] + \omega^2 [J_e + J_e^T] = 0
\]

The nontrivial solutions of Eq. (16) exist only when the determinant of the coefficient matrix of \( u_e \) becomes zero, which will then provide us the following equation for solving natural frequencies:

\[
\frac{1}{2} [K_e + K_e^T] + \omega^2 [M_e + M_e^T] = 0
\]

Based upon (15) and (17) for divergence and free vibration problems, several different problems have been solved numerically in this year. Figures 3 and 4 show two representative results for the divergence and free vibration problems. Detailed explanation of these results can be found in (Yu and Hwu, 2007).
3. Lift of Flapping Wings

In this research, the lifting pressure is approximated by a two-variable second order polynomial, which may be written as

\[ p(x, y) = a\bar{x}^2 + b\bar{y}^2 + c\bar{x}\bar{y} + d\bar{x} + e\bar{y} + f \]  \hspace{1cm} (18)

where \( \bar{x} \) and \( \bar{y} \) are nondimensionalized coordinates defined by \( \bar{x} = x/L \) and \( \bar{y} = y/H \). \( L \) and \( H \) denotes respectively, the length and width of the flapping wings. The coefficients \( a, b, c, d, e, f \) defining the surface variation of the lift will be determined through a well trained artificial neural network (ANN). To provide a well trained ANN, in this research we prepare the training pattern through the finite element modeling. In this modeling the flapping wings are analyzed by using the commercial finite element code ANSYS. The SHELL91 and SHELL99 elements are selected to model the wing skins and skeletons. The skins are made by LD-PE plastics and the skeletons are made by Carbon/Epoxy prepregs. Mapped mesh and free mesh are used in modeling. The wings are fixed at the connection points of wings and flapping mechanism. The convergence analysis is performed to make sure the correctness of the output values. In this network the input vector contains the strain values of three different positions of the wings, while the output vector contains the coefficients of equation (18). To make sure the ANN is well trained, test patterns are prepared through both numerical modeling and experimental measurement. After ANN is well trained, the lift acting on the flapping wings can then be estimated through the strains measured on the wings. Figure 5 shows the mechanism of the flapping wings and three positions for the strain measurement. Figures 6a-c show the lift estimated by ANN and measured strains associated with three different flapping frequencies.
4. Van der Waals force modelling

In the first year of this project, a finite element modeling has been proposed to estimate the Young’s moduli of single-walled carbon nanotubes (SWCNT). In this year, to extend our concept to the modeling of multi-walled carbon nanotubes (MWCNT), we focused on the modeling of Van der Waals force which acts between walls.

An MWCNT is composed of coaxial SWCNTs of different radii. The interactions among these SWCNTs are mainly due to the van der Waals force. In our modeling of MWCNTs, each SWCNT is still simulated as a frame-like structure. The primary bonds between two nearest neighboring atoms are treated as beam members. The cross sections of the beam elements are assumed to be identical and circular. The Young’s modulus $E$ and diameter $d$ of the circular beam can be calculated through the following two formulas (the results of our first year project):

$$ E = \frac{Lk_r^2}{4\pi k_\theta}, \quad d = 4 \sqrt[4]{\frac{k_\theta}{k_r}} $$

Through these two formulas and the known values of bond stretching force constant $k_r$ and bond angle bending force constant $k_\theta$, and C-C bond length $L$, the numerical values of $E$ and $d$ have been calculated as 5.49 Tpa and 0.147 nm respectively. As to the Poisson’s ratio $\nu$, several values of 0.05, 0.1, 0.2 and 0.3 have been examined for the present FE model, and the tests prove that $\nu$ has little effect on the final result in our computation. Therefore, we set $\nu = 0.3$ as a representative value in the FE model.

The van der Waals force between inter-layer is treated as spring element. The stiffness of the spring elements is derived from the equivalent force concept between the inter-layer pressure and the spring forces under the assumption of small deformation such that the variation of the van der Waals force is confined to the linear region. The pressure/inter-layer-distance relation is

$$ p(r) = \frac{\Psi}{6} \left( \frac{r_i}{r} \right)^{10} - \left( \frac{r_o}{r} \right)^4 \right] $$

where $r$ is inter-layer distance, $p$ is the pressure, $r_o$ is the equilibrium distance and $\Psi = 36.5$ GPa. Considering the range (Figure 7) $-1.6 \times 10^{-2} \leq r - r_o \leq 1.6 \times 10^{-2}$, equation (20) can now be approximated by

$$ p(r) = a_o (r - r_o) = a_o \cdot \Delta r $$

where $a_o = -86.5$ GPa/nm is the slope of this linear region, $\Delta r$ is elongation of inter-layer distance $r$.

With the approximated linear relation (21), the spring constants used to simulate the van der Waals force can then be calculated by

$$ k_r = -\frac{a_o d}{N} $$

where $N$ is the number of spring elements and $A$ is the area under the action of van der Waals force.

Figure 8 is a plot showing the modeling of van der Waals force. Figure 9 is the FEM modeling of MWCNT which will be used in our next year.
Figure 8. Representative spring element of van der Waals force

Figure 9. FEM modeling of MWCNT

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References


計畫成果自評
本期計劃預計完成之工作項目有 1. 偶合邊界元素程式之設計；2. 渐縮複材機翼有限元程式設計；3. 拍翼式微飛機之翼表面升力-等效靜態分佈力；4. 凡得瓦力之結構力學模擬。截至目前為止皆已順利完成，亦有相關成果送至國際會議發表，頗受好評，唯成果尚未臻完整階段，目前仍未投稿至國際期刊，依個人之經驗，在半年內應可完成 3-4 篇期刊論文之撰稿工作。