Failure Analysis of Interface Corners (1/3)

1. Introduction

Electric devices are composed of many different parts. Because each part may be made from different materials and may have different shapes, it is very possible that many interface corners exist in several local fields of an electric device. Due to the mismatch of thermal or elastic properties, stress singularity usually occurs near the interface corners, which may initiate failure of structures. Therefore, it is important to design a proper joint shape to prevent the failure initiation and propagation. The singular order of stresses near the interface corners is a good index for the understanding of failure initiation. However, in engineering applications one usually feels only the knowledge of singular orders is not enough for the prediction of failure initiation. The most apparent examples are homogeneous cracks whose singular order is $-1/2$ which is a constant value and is nothing to do with the surrounding environment and outside loading of cracks. These influential factors are reflected through another important parameter – stress intensity factor. Therefore, in
addition to the singular orders one is always interested to know their associated stress intensity factors of interface corners.

Although several detailed studies have been done about the determination of the associated singular orders and stress intensity factors for interface corners, very few failure criteria were successfully established based upon these parameters. Even the cracks in homogeneous media or the cracks lying along the interface between dissimilar materials are the special cases of the interface corners, the definitions of stress intensity factors proposed in the literature are usually not consistent with that of cracks. Therefore, to have a universal failure criterion for the homogeneous cracks, interface cracks and interface corners, a unified definition for the stress intensity factors is indispensable. In the literature the stress intensity factors of interface corners are usually defined by the way similar to the homogeneous cracks, e.g., (Sinclair, et al., 1984; Dunn, et al., 1997), which may encounter trouble when the stress distributions near the interface corners exhibit the oscillatory characteristics like the interface cracks discussed in (Rice, 1988; Wu, 1990; Suo, 1990; Gao, et al., 1992; Hwu, 1993). Thus, even for the interface cracks some definitions of their stress intensity factors proposed in the literature are not compatible with the conventional definitions for homogeneous cracks.

To build a direct connection among the homogeneous cracks, interface cracks and interface corners, in this project a unified definition for the stress intensity factors is proposed.

By the definition of the stress intensity factors proposed in this project, to calculate their values we need to know the stresses near the tip of interface corners. By employing the Stroh formalism for anisotropic elasticity (Ting, 1996), the near tip solutions for elastic composite wedges have been obtained (Hwu, et al., 2004; Hwu and Lee, 2004). Based upon the analytical solutions obtained from (Hwu and Lee, 2004), in this report the near tip solutions for the general interface corners are divided into five different categories depending on whether the singular order is distinct or repeated, real or complex. However, due to the singular and possibly oscillatory behaviors of the near tip solutions, it is not easy to get convergent values for the stress intensity factors directly from the definition. To overcome this problem, a path-independent integral will be considered in our follow-up project.

2. Near tip solutions for interface corners

To study the singular behavior of interface corners and to provide a proper definition for their associated stress intensity factors, like the concept of fracture mechanics it is important to know the near tip solutions. By employing Stroh formalism for anisotropic elasticity, the near tip solutions for multi-bonded anisotropic wedges have been obtained as (Hwu, et al., 2003; Hwu and Lee, 2004)

\[ \mathbf{w}_k(r, \theta) = r^k \mathbf{E}_k(\theta) \mathbf{K}_{k-1} \mathbf{w}_0, \quad k = 1, 2, 3, ..., n, \]

in which \((r, \theta)\) is the polar coordinate with origin located on the wedge apex; \(\mathbf{w}_k(r, \theta), k = 1, 2, ..., n\), is a 6×1 vector composed of the displacements and stress functions of the \(k\)th wedge; \(\mathbf{w}_0\) is a 6×1 coefficient vector related to \(\mathbf{w}_k(r, \theta)\) by \(\mathbf{w}_k(r, \theta) = r^k \mathbf{E}_k(\theta) \mathbf{w}_0\); \(\mathbf{E}_k(\theta)\) and \(\mathbf{K}_{k-1}\) are 6×6 matrices related to the material properties of the wedges. They are defined by (Note that for simplicity the symbols \(1-\delta\) and \(\mathbf{K}_{k-1}\) used in (Hwu and Lee, 2004) has been replaced by \(\lambda\) and \(\mathbf{K}_{k-1}\))

\[
\mathbf{w}_0 = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{\phi}_1 \end{bmatrix}, \quad \mathbf{w}_k(r, \theta) = \begin{bmatrix} \mathbf{u}_k(r, \theta) \\ \mathbf{\phi}_k(r, \theta) \end{bmatrix},
\]

\[
\mathbf{u}_k(r, \theta) = \begin{bmatrix} u_{1,i}(r, \theta) \\ u_{1,j}(r, \theta) \\ u_{1,k}(r, \theta) \end{bmatrix}, \quad \mathbf{\phi}_k(r, \theta) = \begin{bmatrix} \phi_{1,i}(r, \theta) \\ \phi_{1,j}(r, \theta) \\ \phi_{1,k}(r, \theta) \end{bmatrix}.
\]

\[
\mathbf{E}_k(\theta) = \mathbf{N}_k(\theta, \theta_{k-1}), \quad k = 1, 2, 3, ..., n,
\]

\[
\mathbf{K}_{k-1} = \prod_{i=1}^{k-1} \mathbf{E}_i \mathbf{E}_{k-2} \cdots \mathbf{E}_1, \quad k = 2, 3, ..., n,
\]

in which \(\mathbf{E}_k = \mathbf{E}_k(\theta_k) = \mathbf{N}_k(\theta_k, \theta_{k-1})\), and \(\theta_k\), \(\theta_{k-1}\) are the angular location of the two sides of the \(k\)th wedge (Figure 1(a)), \(u_{1,i} = 1, 2, 3, \) are the displacements in \(x_i\) -directions, \(\phi_i, j = 1, 2, 3, \) are the stress functions related to the Cartesian stress components \(\sigma_{ij}\) and surface traction vector \(\mathbf{t}\) by

\[
\sigma_{ij} = -\phi_i, j, \quad \sigma_{ij} = \phi_i, j, \quad \mathbf{t} = \partial \phi_i / \partial s, \quad s \]

where \(s\) is the arc length measured along the curved boundary such that when one faces the direction of increasing \(s\) the material lies on the
right side. From (3b), we have \( \mathbf{t} = \hat{\mathbf{\phi}}_r \) for a radial line surface and \( \mathbf{t} = \hat{\mathbf{\phi}}_f / r \) for a circular surface, and hence, the stresses in polar coordinate can also be calculated from the stress functions \( \hat{\mathbf{\phi}} \) by

\[
\begin{align*}
\sigma_{\theta\theta} &= \mathbf{n}^T \hat{\mathbf{\phi}}_r, \\
\sigma_{rr} &= -\mathbf{n}^T \hat{\mathbf{\phi}}_f / r, \\
\sigma_{\theta r} &= \mathbf{n}^T \hat{\mathbf{\phi}}_f / r, \\
\sigma_{\theta r} &= -\mathbf{n}^T \hat{\mathbf{\phi}}_f / r,
\end{align*}
\]

(4a)

where

\( \mathbf{n}^T = (\cos \theta \ \sin \theta \ 0), \quad \mathbf{m}^T = (-\sin \theta \ \cos \theta \ 0) \)  

(4b)

In (2), \( \hat{\mathbf{N}} \) is a \( 6 \times 6 \) matrix related to the Stroh fundamental matrix \( \mathbf{N} \) (Ting, 1996) by

\[
\hat{\mathbf{N}}^i(\theta, \theta_{-1}) = \left[ \cos(\theta - \theta_{-1}) \mathbf{I} + \sin(\theta - \theta_{-1}) \mathbf{N}^i(\theta_{-1}) \right]^T
\]

(5)
in which \( \mathbf{I} \) is a \( 6 \times 6 \) identity matrix. Because in general \( \lambda \) is not an integer, to calculate the \( \lambda \) power of \( \hat{\mathbf{N}} \) one usually use the transformation through the eigenvalues and eigenvectors of \( \mathbf{N} \). By this way, it has been proved that (Hwu, et al., 2003)

\[
\hat{\mathbf{N}}^i(\theta, \theta_{-1}) = \left[ \hat{\mathbf{A}}_i \hat{\mathbf{A}}_i^T < \hat{\mu}_i^j(\theta, \theta_{-1}) > 0 \right] \mathbf{B}_i \mathbf{B}_i^T < \hat{\mu}_i^j(\theta, \theta_{-1}) > \left[ \hat{\mathbf{B}}_i \hat{\mathbf{B}}_i^T \right]^T
\]

(6)
in which the overbar denotes the complex conjugate; the angular bracket \( < > \) stands for a diagonal matrix in which each component is varied according to the subscript \( * \), e.g., \( < z > = \text{diag}[z_1, z_2, z_3] \); the superscript \( T \) denotes the transpose of a matrix. \( \mathbf{A} \) and \( \mathbf{B} \) are two \( 3 \times 3 \) material eigenvector matrices and \( \hat{\mu}_i^j(\theta, \theta_{-1}) \) is related to the material eigenvalues \( \mu_i \) by

\[
\hat{\mu}_i^j(\theta, \theta_{-1}) = \left[ \cos(\theta - \theta_{-1}) + \sin(\theta - \theta_{-1}) \mu_i(\theta_{-1}) \right]^T, \quad *, = 1, 2, 3,
\]

(7a)

and

\[
\mu_i(\theta_{-1}) = \frac{\mu_i \cos \theta_{-1} - \sin \theta_{-1}}{\mu_i \sin \theta_{-1} + \cos \theta_{-1}}.
\]

(7b)

In the above, \( \lambda - 1 \) is the order of the stress singularity which will be influenced by the wedge configurations (n wedge angles) and properties (21n elastic constants), and the boundary conditions of wedge surfaces. For free-free wedge \( \hat{\mathbf{\phi}}_f(\rho, \theta_f) = \hat{\mathbf{\phi}}_f(\rho, \theta_a) = 0 \), detailed derivation will lead to (Hwu and Lee, 2004)

\[
\hat{\mathbf{\phi}}_0 = 0, \quad \mathbf{K}^{(3)} \mathbf{u}_0 = 0 \quad \text{(8a)}
\]

where \( \mathbf{K}^{(3)} \) is one of the submatrices of \( \mathbf{K} \) which is related to \( \mathbf{E}_k \) by

\[
\mathbf{K} = \prod_{k=1}^n \mathbf{E}_{n-k+1} \mathbf{E}_{n-1} \ldots \mathbf{E}_1, \quad \mathbf{K} = \left[ \mathbf{K}^{(1)} \mathbf{K}^{(2)} \right]
\]

(8b)

Nontrivial solution of \( \mathbf{u}_0 \) exists only when the determinant of \( \mathbf{K}^{(3)} \) is equal to zero, i.e.,

\[
\| \mathbf{K}^{(3)} \| = 0, \quad \text{which will give us the singular order } \lambda - 1.
\]

After obtaining the singular orders that may be real or complex, distinct or repeated, the nonzero values of \( \mathbf{u}_0 \) can then be calculated through (8a). With \( \lambda \) and \( \mathbf{w}_o = (\mathbf{u}_0)^T \) determined, the near tip solution (1) can now be expanded as

\[
\mathbf{u}_k(r, \phi) = r^{\lambda} \mathbf{E}_k^{(1)}(\phi) \mathbf{K}^{(1)}_k + r^{\lambda} \mathbf{E}_k^{(2)}(\phi) \mathbf{K}^{(2)}_k \mathbf{u}_o,
\]

(9a)

\[
\hat{\mathbf{\phi}}_k(r, \phi) = r^{\lambda} \mathbf{E}_k^{(3)}(\phi) \mathbf{K}^{(3)}_k + r^{\lambda} \mathbf{E}_k^{(4)}(\phi) \mathbf{K}^{(4)}_k \mathbf{u}_o,
\]

(9b)

where \( \mathbf{E}_k^{(i)}(\phi) \) and \( \mathbf{K}^{(i)}_k \) are the submatrices of \( \mathbf{E}_k^{(i)}(\phi) \) and \( \mathbf{K}^{(i)}_k \), defined by

From (8a) and the definitions given in (2) and (5)-(7), we see that the singular orders are totally determined through the material properties and configurations of all wedges. Since we consider the singular fields and the strain energy cannot be unbounded, only the values located in the range of \( 0 < \text{Re}(\lambda) < 1 \) are considered in this report. If more than one \( \lambda \) locate in this range, we select the one whose real part is minimum as \( \lambda_c \), i.e., the one with the most critical singular order \( \lambda - 1 \). If \( \lambda_c \) is a complex number, it has been proved that its conjugate \( \bar{\lambda}_c \) is also a root of

\[
\| \mathbf{K}^{(3)} \| = 0 \quad \text{(Hwu, et al., 2003)}.
\]

When \( r \to 0 \), i.e., the near tip field, the terms associated with \( \lambda_c \) will dominate the stress behavior. Neglecting all the other singular and nonsingular terms, and expanding the near tip solution (1) for the terms associated with \( \lambda_c \), we may express the displacement and stress function vectors in terms of the eigenvector \( \mathbf{u}_o \) obtained from (8a). However, an eigenvalue \( \lambda_c \) may correspond several linearly independent eigenvectors \( \mathbf{u}_o \). If \( \lambda_c \) is a non-repeated root, only one arbitrary scalar is needed to describe \( \mathbf{u}_o \).
When \( \lambda_c \) is a double root, two arbitrary scalars are needed. While for a triple root \( \lambda_c \), three arbitrary scalars are needed. If \( \lambda_c \) is complex, the arbitrary scalar associated with \( u_c \) is also complex which contains two real scalars. With the above understanding, the near tip solutions (9a) may now be rewritten as:

**Case 1:** \( \lambda_c \) is distinct and real, \( \lambda_c = \lambda_g \),

\[
\mathbf{u}(r, \theta) = e^{-i \theta} \mathbf{p}(\theta),
\]

\[
\phi(r, \theta) = e^{-i \theta} \mathbf{q}(\theta),
\]

(10a)

**Case 2:** \( \lambda_c \) is double and real, \( \lambda_c = \lambda_g \),

\[
\mathbf{u}(r, \theta) = e^{-i \theta} \{ c_c \mathbf{p}_1(\theta) + c_t \mathbf{p}_2(\theta) \},
\]

\[
\phi(r, \theta) = e^{-i \theta} \{ c_c \mathbf{q}_1(\theta) + c_t \mathbf{q}_2(\theta) \},
\]

(10b)

**Case 3:** \( \lambda_c \) is triple and real, \( \lambda_c = \lambda_g \),

\[
\mathbf{u}(r, \theta) = e^{-i \theta} \{ c_c \mathbf{p}_1(\theta) + c_t \mathbf{p}_2(\theta) + c_{tt} \mathbf{p}_3(\theta) \},
\]

\[
\phi(r, \theta) = e^{-i \theta} \{ c_c \mathbf{q}_1(\theta) + c_t \mathbf{q}_2(\theta) + c_{tt} \mathbf{q}_3(\theta) \}.
\]

(10c)

**Case 4:** \( \lambda_c \) is distinct and complex, \( \lambda_c = \lambda_g \pm i \epsilon \),

\[
\mathbf{u}(r, \theta) = e^{-i \theta} \{ c_c \mathbf{p}_1(\theta) + e^{-i \epsilon} \mathbf{p}(\theta) \},
\]

\[
\phi(r, \theta) = e^{-i \theta} \{ c_c \mathbf{q}_1(\theta) + e^{-i \epsilon} \mathbf{q}(\theta) \}.
\]

(10d)

**Case 5:** one is real \( \lambda_g \) and the others are complex \( \lambda_c = \lambda_g \pm i \epsilon \),

\[
\mathbf{u}(r, \theta) = e^{-i \theta} \{ c_c \mathbf{p}_1(\theta) + e^{-i \epsilon} \mathbf{p}(\theta) \},
\]

\[
\phi(r, \theta) = e^{-i \theta} \{ c_c \mathbf{q}_1(\theta) + e^{-i \epsilon} \mathbf{q}(\theta) \}.
\]

(10e)

In the above, \( \mathbf{p}(\theta) \) and \( \mathbf{q}(\theta) \) (or \( \mathbf{p}_i(\theta) \) and \( \mathbf{q}_i(\theta) \), \( i = 1, 2, 3 \)) are functions related to \( F^{(i)}(\theta), K^{(i)}_{\theta \theta} \) and \( u_0 \) in which the number of arbitrary scalars is defined on the multiplicity of \( \lambda_c \). Note that the solutions shown in (10a-e) are valid for any wedge of the multibonded wedges, and hence from now on unless special notification is needed the subscript \( k \) denoting the wedge has been neglected for simplicity.

### 3. A unified definition for stress intensity factors

It is known that a semi-infinite crack in homogeneous materials can be represented by letting \( \theta_0 = -\pi \) and \( \theta_1 = \pi \) for a single wedge. Moreover, an interface crack can be represented by a bi-wedge with \( \theta_0 = -\pi \), \( \theta_1 = 0 \) and \( \theta_2 = \pi \). These two important special cases indicate that to propose a proper definition for the stress intensity factors of interface corners, it is better to review the corresponding definition for the crack problems.

A conventional definition for the stress intensity factors \( k \) of a crack in homogeneous media is (Broek, 1974)

\[
k = \begin{cases}
K_{II} & \frac{\partial}{\partial \theta} \sigma_{\theta \theta} = \lim_{r \to 0} \sqrt{2\pi r} \sigma_{\theta \theta}, \\
K_{III} & \frac{\partial}{\partial \theta} \sigma_{\theta \theta} = \lim_{r \to 0} \sqrt{2\pi r} \phi_{\theta}, \\
K_{I} & \frac{\partial}{\partial \theta} \sigma_{\theta \theta} = \lim_{r \to 0} \sqrt{2\pi r} \sigma_{\theta \theta},
\end{cases}
\]

(11)

in which the third equality of (11) comes from the relations given in (4); \( \theta = 0 \) is a line along the crack. Due to the oscillatory behavior of the stresses near the tip of interface cracks, this definition cannot be applied to the cracks lying on the bimaterial interface. A proper definition for the bimaterial stress intensity factors has been given by Hwu (1993) as

\[
k = \left\{ \begin{array}{l}
K_{II} = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \sigma_{\theta \theta}, \\
K_{III} = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \phi_{\theta}, \\
K_{I} = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \phi_{\theta},
\end{array} \right.
\]

(12a)

or in matrix form

\[
k = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \phi_{\theta},
\]

(12b)

where \( \Lambda = \left[ \mathbf{q}_i, \tilde{\mathbf{q}}_i, \mathbf{q}_i, \right] \), \( \epsilon, \tilde{\epsilon}, \epsilon_* = 1, 2, 3 \), are the eigenvalues and eigenvectors of \( \mathbf{M}^* = e^{2\pi i \mathbf{M}} \mathbf{q} = 0 \),

(13a)

in which \( \mathbf{M}^* \) is the bimaterial matrix related to the material eigenvector matrices \( \mathbf{A}_i, \mathbf{B}_i, i = 1, 2 \) (Hwu, 1993; Ting, 1996). In (3.2), \( \ell \) is a length parameter which may be chosen arbitrarily as long as it is held fixed when specimens of a given material pair are compared. Different values of \( \ell \) will not alter the magnitude of \( k \) but will change its phase angle. In application, the reference length \( \ell \) is usually selected to be the crack length.

Combining (11) and (12), and considering the consistence of definitions, we now propose a unified definition for the stress intensity factors of interface corners, which is also applicable for the cracks in homogeneous materials or bimaterial interfaces, as

\[
k = \left\{ \begin{array}{l}
K_{II} = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \sigma_{\theta \theta}, \\
K_{III} = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \phi_{\theta}, \\
K_{I} = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \phi_{\theta},
\end{array} \right.
\]

(14a)

or in matrix form

\[
k = \lim_{r \to 0} \sqrt{2\pi r} \Lambda^{-1} \phi_{\theta},
\]

(14b)

in which \( \Lambda \) is a matrix related to the wedge
configurations and properties. In general, if $\lambda_i$ is real, i.e., $\varepsilon_i = 0$, $\Lambda$ is not required for the definition of $k$ since $\Lambda < (r/\ell)^{-i\varepsilon} > \Lambda^{-1}$ in (14) is equal to the identity matrix $I$. With this understanding, only cases 4 and 5 shown in (10d,e) need a proper definition for the matrix $\Lambda$.

Differentiating the stress function vector $\phi(r,0)$ with respect to $r$ for each case shown in (10a-e), and substituting the results into (14b), we can get the relations between the coefficients $c_i$ (or simply $c$) and the stress intensity factors $k$ as follows.

Case 1:
$$k = \sqrt{2\pi c_\lambda q(0)}.$$  
(15a)

Case 2:
$$k = \sqrt{2\pi \lambda_{21} A^c},$$

where $\Lambda^{*} = [q_0(0), q_1(0)]$, $c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.  
(15b)

Case 3:
$$k = \sqrt{2\pi \lambda_{21} A^c},$$

where $\Lambda^{*} = [q_0(0), q_1(0), q_2(0)]$, $c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$.  
(15c)

Case 4:
$$k^* = \sqrt{2\pi \Lambda < (\lambda^2 + i\varepsilon_{1})^{i\varepsilon} > c} ,$$

where $\Lambda = \left[ q'(0), q''(0) \right]$, $c = \begin{bmatrix} c \\ \tau \end{bmatrix}$.  
(15d)

Case 5:
$$k = \sqrt{2\pi \Lambda < (\lambda^2 + i\varepsilon_{1})^{i\varepsilon} > c} ,$$

where $\Lambda = \left[ q(0), q''(0), q_3(0) \right]$, $c = \begin{bmatrix} c \\ \tau \end{bmatrix}$.  
(15e)

Note that in case 2 $\Lambda^{*}$ is a $3 \times 2$ matrix, while in case 3 $\Lambda^{*}$ is a $3 \times 3$ matrix. In case 4 $k^*$ is a $2 \times 1$ vector that could be $(K_y, K_z)^T$ or $(K_y, K_{32})^T$ depending on the contents of the $2 \times 1$ vector $q''(0)$ which may contain the first two components or the last two components of the $3 \times 1$ vector $q(0)$. The associated diagonal matrix denoted by $<>$ shown in (15d) is a $2 \times 2$ matrix in which each component is varied according to the subscript $* = 1,2$, and $\varepsilon_1 = \varepsilon, \varepsilon_2 = -\varepsilon$. While in case 5, the associated diagonal matrix is a $3 \times 3$ matrix in which each component is varied according to the subscript $* = 1,2,3$ and $\varepsilon_1 = \varepsilon, \varepsilon_2 = -\varepsilon, \varepsilon_3 = 0$. Thus, $\Lambda$ is a $2 \times 2$ matrix in case 4 and a $3 \times 3$ matrix in case 5.

A general matrix form for the relations shown in (15a-e) may be written as
$$k = \sqrt{2\pi \Lambda < (\lambda^2 + i\varepsilon_{1})^{i\varepsilon} > c} ,$$  
(16)
in which the contents of matrices $k$, $\Lambda$ and $c$ depend on whether the singular order is distinct or repeated, real or complex as those described in (10).

From the above discussion, we see that the definition for the stress intensity factors proposed in (14) is applicable not only to the interface corners but also to the cracks in homogeneous media or bimaterial interfaces. The conventional definition (11) is just a special case of (14) with $\lambda_{21} = 1/2$ and $\varepsilon = 0$, while the definition for the bimaterial stress intensity factor (12) is a special case of (14) with $\lambda_{21} = 1/2$. With this unified definition, it becomes possible that the failure criteria developed for the crack problems may be useful for the prediction of the failure of interface corners. Moreover, the fracture toughness measured from the standard crack specimen may also have a direct connection with the toughness of interface corners.

It should be noted that the unified definition proposed in (14) is valid only for the most critical singular order $\lambda = -1$. For the cases that two or more but different eigenvalues exist in the range $0 < \text{Re}(\lambda_i) < 1$, definition (14) cannot provide meaningful constant factors for the lower critical singular orders. For example, if $\phi' \rightarrow c_1 \lambda_{21} \lambda^{-1} - q_1(\theta) + c_2 \lambda_{32} r^{i\varepsilon} - q_1(\theta)$ when $r \rightarrow 0$, no constant value of $k_i$ related to $\lambda_i$ can be got through the unified definition shown in (14). The possible way to get $k_i$ is further modifying the definition with $\phi_1$ replaced by $\phi_1'$ where $\phi_1' = \phi_1 - c_1 \lambda_{21} \lambda^{-1} - q_1(\theta)$, which means that the near tip stresses of (14a) should be subtracted by the dominant portion of $\lambda_{21} = -1$. Since this will make the unified definition more complicated than before, in this project we just focus on the stress intensity factors of the most critical singular order.

Acknowledgements

The principal investigator (PI) would like to thank the support by National Science Council, Republic of China, through Grant No. NSC 95-2221-E-006-144-MY3.
References


計畫成果自評

本期計畫運用界面角與裂縫問題之連接關係，尋找可同時適用於裂縫問題與界面角問題之應力強度因子，為定義一通用之應力強度因子，在本期計畫中我們先嘗試將界面角尖端之應力場及位移場以較簡潔明確且通用之方式表示，結果一如我們的預期，已順利定義出一同適用於一般裂縫、界面裂縫與界面角問題之通用應力強度因子。相關成果亦著手整理投稿國際期刊中。