On-Line Measurement and Optimal Control of Composite Wing Structures (1/3)

Abstract

A comprehensive model for the stiffened composite multicell wing structures has been developed in our previous project. In this model, the extensional, bending and coupling stiffnesses are calculated not only from the properties of the composite laminates of the wing skin but also from the stringers and spar flanges which are treated as fibers of a pseudo-lamina. The arrangement of the wing spar webs and ribs is then treated like a sandwich honeycomb core, from which an equivalent transverse shear modulus of the wing structures is calculated. With these estimated properties, the composite wing structure is modeled incorporating the effects of bending-torsion coupling, warping restraint, transverse shear deformation, shape of airfoil, rotary inertia, etc. To avoid the complexity of formulation, the matrix form representation was introduced in our previous study, which then makes most of the equations bear the same form as those of the classical beam theory. Thus, due to the similarity of the matrix form representation, it is very possible that the composite wing structures can be studied analytically by following the classical approach. Therefore, in this project, we go further on free and forced vibration of composite wing structures and also verify our results by the commercial finite element software ANSYS. In addition, we also work on the improvement of neural network. These results all become strong fundamental backgrounds of our following studies such as on-line measurement of material properties, vibration suppression, optimal design, optimal placement of sensors and actuators, etc.

Keywords: Composite Wing Structures, Vibration Analysis, On-Line Measurement, Vibration Suppression, Optimal Design, Sensors, Actuators

1. Introduction

Over the last five decades several different structural models have been proposed to study the composite wing structures such as the classical beam model [1,2], the coupled bending-torsion beam model or called box beam model [3-5], and the refined model considering the warping restraint [6-9] and/or transverse shear deformation [10] and/or shell bending strain [11, 12] and/or cross-sectional materials and geometries [13,14], etc. [15-18]. A review of such literature prior to 1988 can be found in [19], whereas the most recent review about the composite beam modeling has been given by Jung, et al. [20] and Voloioi, et al. [21]. To have an improved prediction for the mechanical behavior of the stiffened composite multicell wing structures, the comprehensive refined model incorporating the various elastic and structural couplings, the warping restraint, the transverse shear deformation, the shape of airfoil effects and so on should naturally be a better choice for the structural analysis. However, it is expectable that the more comprehensive the model is, the more difficult to get meaningful and useful analytical results. Hence even the comprehensive model was proposed around ten years ago, most of the analytical works of wing structural analysis still used the box beam model if the ignored effects will not induce drastic change of the results. In order to benefit from the accuracy of the
comprehensive model as well as the simplicity of the classical beam model, in this project we re-derive the comprehensive model into a simple and elegant form by using the matrix form representation. Due to its simplicity, the vibration analysis by the comprehensive model, which is expected to be complicated, now becomes as simple as that of classical beam model.

To show the accuracy and generality of the comprehensive dynamic model for the vibration analysis of the stiffened composite multicell wing structures, several illustrative examples are given such as a bending-torsion coupled cantilever composite beam, a cantilever composite sandwich beam, a NASA 2412 composite wing. Moreover, the influential factors on the natural frequencies, such as the transverse shear deformation, warping restraint, shape of airfoil are all studied by this unified and general formulation. An example for the forced vibration analysis is then given by the vibration suppression of composite wing with piezoelectric sensors and actuators bonded on the wing surfaces, for which a LQG/LTR controller is designed and proved to be successful for vibration suppression of composite wings.

In addition to the vibration analysis of composite wing structures, for the study of on-line measurement of next year in this year we also work on the improvement of neural network.

2. Matrix Form Comprehensive Model

The primary function of the wing structure is providing the lift for an aircraft, which is governed by the aerodynamic consideration. In addition to the aerodynamic pressure, there are other forces resisted by the wing structures such as the weight of the structures, fuels, engines, undercarriage system, and possibly carried weapons, and the thrust of engines, etc. To sustain these loads, the wing structures usually consist of axial members in stringers, bending members in spars, shear panels in the cover skin and spar webs, and planar members in ribs. If the cover skin of the wing is made of the composite laminates, the entire wing structure may be simulated by a composite sandwich plate in which the wing skins and stringers (including the spar flanges) are simulated as the faces while the spar webs and ribs are simulated as the core of the sandwiches. Because the wing cross section must have a streamlined shape commonly referred to as an airfoil section, the thickness of the sandwich will not be a constant but a function of the airfoil. Moreover, as the usual sandwich assumptions, the thickness is not too small to neglect the transverse shear deformation. Based upon these considerations, a mathematical model for the composite sandwich plates [22,23] has been applied by Hwu and Tsai [18] to model the stiffened composite multicell wing structures.

Due to the closely spaced stringers and the transverse stiffening members like wing spars and ribs, in aircraft analysis it is usually assumed that the wing chordwise section is rigid [2]. The displacement field consistent with the chordwise-rigid postulation and the basic assumptions for the composite sandwich plates can be written as [18]

\[
\begin{align*}
    u(x,y,z,t) &= z \theta(y,t), \\
    v(x,y,z,t) &= v_0(y,t) + z(\beta_y(y,t) + x \beta_r(y,t)), \\
    w(x,y,z,t) &= w_0(y,t) - x \theta(y,t),
\end{align*}
\]

where \(u, v, w\) are the displacement components in the directions of \(x\) (chordwise), \(y\) (spanwise) and \(z\) (thicknesswise), respectively. \(t\) denotes the time variable. \(v_0\) is the mid-plane displacements in \(y\) direction. \(w_0\) denotes the deflection (positive upward) measured at the line of the reference axis; \(\theta\) is the rotation angle with respect to \(x\)-axis due to the twist around the reference axis (positive nose up), i.e., \(\beta_y = \theta\). \(\beta_r\) denotes the rotation angle with respect to \(y\) axis measured at the reference axis and \(\beta_r\) stands for the rate of angle change in the \(x\)-direction. Thus, \(\beta_r = \beta_r + x \beta_r\).

According to the postulation given in (1), the basic functions describing the deformation of the stiffened composite wing structures become \(v_0, w_0, \theta, \beta_y, \beta_r\). Based upon the assumed displacement field, the equations of motion for composite sandwich plates and the constitutive relations for laminated sandwiches, a matrix form comprehensive model has been established [24]. The governing equation for the stiffened composite wing structures in terms of the basic function vector \(\delta\) can be written as

\[
\begin{align*}
    \mathbf{K}_0 \delta'(y,t) + (\mathbf{K}_1 - \mathbf{K}_0) \delta'(y,t) - \mathbf{K}_0 \delta(y,t) + p(y,t) = \mathbf{I}_0 \delta(y,t),
\end{align*}
\]

where

\[
\begin{bmatrix}
    \tilde{\mathbf{p}}_y \\
    \tilde{\mathbf{p}}_r \\
    m_x - \tilde{\mathbf{p}}_r \\
    \tilde{m}_y \\
    \tilde{m}_r
\end{bmatrix}
= \begin{bmatrix}
    v_0 \\
    w_0 \\
    \beta_y \\
    \beta_r
\end{bmatrix},
\]

and

\[
\begin{align*}
    \mathbf{p} = \mathbf{p}(y,t)
\end{align*}
\]
If the wing is fixed at the root \((y = 0)\) and free at the tip \((y = L)\), the boundary conditions of the wing structures can be written as
\[
\mathbf{a}(0) = 0, \quad \text{and} \quad \mathbf{K}_0 \mathbf{a}(L) + \mathbf{K}_1 \mathbf{a}'(L) = 0. \tag{3}
\]

If the influence of the in-plane spanwise surface loads as well as of in-plane and rotary inertia terms can be disregarded, the 10th-order system of equations (2) can be reduced to an equivalent 8th-order system of equations [24]. Because the displacement field assumed in equation (1) is quite general, several special conditions considered in the literature can be covered by the present formulation. For example, (i) neglect of the transverse shear deformation can be formulated by letting \(y_x = 0\) which will lead to \(\beta_r = -w_y, \beta_t = \theta'\); (ii) neglect of the warping restraint effect can be formulated by letting \(e_y = v_y + z\beta_r, \) which will lead to \(\lambda = 0\); (iii) reduction to the conventional composite sandwich beams can be formulated by letting \(\theta = \beta_r = 0\); and (iv) reduction to the conventional laminated beams can be formulated by letting \(\theta = \beta_r = -w_y, \quad I_z = 0\). For those who are interested in the reduction of our general formulations to these special cases, detailed derivations are presented in [24].

3. Vibration Analysis

Free Vibration

To know the natural frequency and its associated vibration mode of the stiffened composite wing structures, we consider the case that the external forces, torsional moment as well as the distributed moments are all zero. To find the natural modes of vibration, the usual way is the method of separation of variables. By this method we write the deflection as a product of a function of the spatial variables only and a function depending on time only. Thus,
\[
\mathbf{a}(y,t) = \mathbf{A}(y)e^{i\omega t}, \tag{4a}
\]

where
\[
\mathbf{A}(y) = \begin{bmatrix} V_y(y) \\ W_y(y) \\ \Theta(y) \\ B_z(y) \\ B_y(y) \end{bmatrix} \tag{4b}
\]

Through the use of (4), the equation of motion (2a) can easily be reduced to a system of ordinary differential equations
\[
\mathbf{K}_2 \mathbf{A}''(y) + (\mathbf{K}_1 - \mathbf{K}_2^2) \mathbf{A}'(y) - \mathbf{K}_2 \mathbf{A}(y) + \omega^2 \mathbf{I}_0 \mathbf{A}(y) = 0, \tag{5}
\]

which can be solved by letting
\[
\mathbf{A}(y) = d e^{i\omega y}. \tag{6}
\]

Substituting (6) into (5), we get
\[
\left[\mathbf{K}_2 e^{2i\omega} + (\mathbf{K}_1 - \mathbf{K}_2^2) e^{i\omega} \mathbf{I}_0 \right] \mathbf{d} = 0, \tag{7}
\]

whose non-vanishing solutions exist only when the determinant of the coefficient matrix of \(d\) becomes zero, i.e.,
\[
\text{det} \left[\mathbf{K}_2 e^{2i\omega} + (\mathbf{K}_1 - \mathbf{K}_2^2) e^{i\omega} \mathbf{I}_0 \right] = 0. \tag{8}
\]

Equation (8) is a 10th-order polynomial which will have 10 roots \(\epsilon_1, \epsilon_2, ..., \epsilon_{10}\). Each of the roots has an associated eigenvector \(\mathbf{d}_i(\omega)\) determined from (7). Linear superposition of these ten homogeneous solutions now gives us
\[
\mathbf{A}(y) = \sum_{i=1}^{10} \mathbf{d}_i \mathbf{e}^{i\epsilon_i y}. \tag{9}
\]

Substituting (9) and (4) into the boundary conditions (3) will then set a system of ten simultaneous linear algebraic equations with ten unknown coefficients \(k_i\) as
\[
\begin{bmatrix} \mathbf{K}_1 \mathbf{D} < e^{i\epsilon_1} > + \mathbf{K}_2 \mathbf{D} < r e^{i\epsilon_r} > \\ \mathbf{D} \end{bmatrix} = 0, \tag{10a}
\]

where
D = [d_1, d_2, ..., d_{10}] \quad k = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_{10} \end{pmatrix} \quad (10b)

and the angular bracket <> stands for a diagonal matrix in which each component is varied according to its subscript i.

Because both of the eigenvalues r_i and the eigenvectors d_i are functions of the natural frequency \omega, the coefficient matrix of k in (10a) is a function of the natural frequency \omega. Again, non-vanishing solutions exist only when the determinant of the coefficient matrix of k becomes zero, by which we can then obtain the natural frequencies of the stiffened composite wing structures. With the determined natural frequency \omega_0, the coefficients \eta_i can be calculated from (10a) as the eigenvector, and hence the natural vibration mode shapes of the composite wing structures are obtained from (9).

Orthogonality Condition:

If the family of natural vibration mode shapes \lambda_i(y) can constitute a complete set of orthonormal modes, most of the vibration problems can be solved by modal analysis through the use of the expansion theorem. Let \omega_i and \omega_j be the two distinct natural frequencies and \lambda_i(y) and \lambda_j(y) be the corresponding natural modes of vibration resulting from the solution of the equations of motion (5) and its associated boundary conditions (3). It can be proved that [24]

\[ \int_0^L \lambda_i^T \lambda_j \, dy = \delta_{ij}, \quad (11) \]

where \delta_{ij} is the Kronecker delta. Unlike the usual orthogonality conditions that only the lateral deflection is considered, the orthogonality found in (11) shows that the complete set includes not only the mode shapes of the deflection but also the mode shapes of all the other basic functions.

Forced Vibration

After finding the orthogonality relation (11), the expansion theorem may be used to obtain the system response by modal analysis. Using the expansion theorem we write the solution of (2a) as a superposition of the natural modes \lambda_i(y) multiplying corresponding time-dependent generalized coordinates \eta_i(t). Hence,

\[ \delta(y, t) = \sum_{j=1}^\infty \lambda_j(y) \eta_j(t). \quad (12) \]

Introducing (12) into (2a) and utilizing the orthogonality relation (11), we obtain an infinite set of uncoupled second order ordinary differential equation system as

\[ \ddot{\eta}_j(t) + \omega_j^2 \eta_j(t) = N_j(t) \quad j = 1, 2, \ldots \quad (13a) \]

where \( N_j(t) \) denotes a generalized force associated with the generalized coordinate \eta_j(t) and is related to the load vector \( p \) by

\[ N_j(t) = \int_0^L \lambda_j^T(y) p(y, t) \, dy. \quad (13b) \]

4. Numerical Examples

Because the displacement fields assumed in (1) are quite general, as far as the authors' knowledge no analytical solution for the vibration analysis has been provided in the literature. However, several special conditions have been discussed vastly in the literature. Therefore, in this project we first check our solutions by comparison with the existing numerical solutions for some special cases which can be reduced from our general formulation. After verification through the known results, we check the assumptions that the in-plane and rotary inertia terms can be disregarded for the cases of “absence of in-plane spanwise loads”. When these basic checks are done, we illustrate numerical results of natural frequencies and mode shapes of a NACA 2412 composite wing. From this case, we study some influential factors such as transverse shear deformation, warping restraint, shape of airfoil, etc. Finally, through the analysis for forced vibration presented in this report, applications to vibration suppression of composite wing are also illustrated. Some of these numerical examples have been shown in [24,25] and some others are still under working for the cases of following topics such as on-line measurement of material properties, optimal placement of sensors and actuators, etc. Figures 1 and 2 are two typical results for the studies of influential factors.

Figure 1: Effect of transverse shear modulus on natural frequency
5. Pre-Work of Neural Network

Back-propagation neural network is widely applied to every field. It usually adopts the steepest descent method to search minimum of objective function. But the steepest descent method requires lengthy training time and is easy to trap into local minimum. To speed up the convergence, in this project we consider four different training methods (conjugate gradient method, Levenberg Marquardt method, HWKO-WO method and conjugate gradient - Levenberg Marquardt method) for back-propagation neural network. By observing the shortcomings and advantages of each method and we try to induce some suggestions for our future study of on-line measurement. Among them conjugate gradient - Levenberg Marquardt method (CGLM method), which combines conjugate gradient method and Levenberg Marquardt method, possesses better convergence than the other methods and is the candidate for our next work. Figure 3 is the flow chart of CGLM method. Detailed discussion of these methods can be found in [26].

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References


