複材翼結構之氣控彈問題(1/3)
Aeroservoelasticity of Composite Wing Structures(1/3)

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中文摘要
近幾年我個人曾針對複合夾心板建立一合理之理論模式，並利用這一模式推導出挫屈和自由振動等問題之閉合解，進而探討糧構構件之最佳化組合。因為糧構構件較易在心材與面材之間發生脫層，吾人亦曾將此一理論模式延伸至含脫層的複合夾心樑，並順利導得相關問題之閉合解。同時唯恐此一閉合解與實際狀況不合，又進行了數值與實驗分析以證實吾人所提閉合解的正確性。經過這一系列的研究後，吾人對所提之複合夾心板理論已有相當程度的信心。因為一般之複材機翼結構，蒙皮(wing skin)部份可視為複合夾心板的複材面板，翼肋(wing rib)、翼樑肋(spar web)及其餘架空部份可視為心材，因此可考慮為一非等厚之複合夾心板，而厚度之變化即翼形之形狀函數。在本計畫的第一年即以吾人所發展之複合夾心板理論為基礎，進行複材機翼結構的模擬工作。目前甫順利完成模擬工作，且比預定的進度提前，已在進行氣彈交互作用產生之發散問題。

關鍵詞：複合夾心板，複材機翼結構，氣控彈分析，發散，顫振，控制

Abstract
Recently, my co-workers and I developed a theoretical model for the mechanical analysis of the composite sandwich plates. Through this model, several analytical closed-form solutions have been obtained, such as the solutions for the buckling loads and natural frequency, etc. Furthermore, an optimal arrangement for the composite sandwich plates has also been made by using these closed-form solutions. Based upon this model, the closed-form solutions for the buckling loads and natural frequency of the delaminated composite sandwich beams have also been obtained. Moreover, several numerical and experimental works have been carried out to ensure that our theoretical results are consistent with the real situations. After this series of research works, we are very confident with our theoretical model. In this proposal we try to model a composite wing structure by using the developed composite sandwich plate model since the skin of a composite wing structure can be treated as the face of a composite sandwich plate, and the wing rib and the spar web and all the vacancy can be treated as the core of a composite sandwich plate. The thickness of the plate is then varied according to the shape function of the airfoil. Based upon this simulation, at the first year we have successfully modelled the composite aircraft structures. Furthermore, we go ahead of schedule to work on the problems on aeroelastic divergence of composite wing structures.

Keywords: Composite Sandwich plates, Composite Wing Structures, Aeroservoelasticity, Divergence, Flutter, Control

1. Introduction
The fundamental work concerning the divergence instability of swept metallic wings was done about fifty years ago 1. It was shown that bending deflections have a destabilizing effect on the swept-forward wings. Hence the swept-forward wing aircraft as a possible option was completely eliminated for a long time, until the aeroelastic tailoring concept for the composite wing structures was raised and studied by Krone 2. Following his work, many different approaches and considerations have been studied, such as the works done by Lerner and Markowitz 3, Weisshaar 4, Oyibo 5, Lottatiru 6, and a series works done by Librescu and his co-workers 7,10. They first considered the warping restraint effects 7,9, then considered the effects of transverse shear strains 10. All these results support that a composite swept-forward wing can be tailored to overcome this adverse instability phenomenon.

Most of the governing equations provided in the literature are obtained from the Hamilton’s variational principle. Due to the complexity of the formulation, it is difficult to see any physical indication purely from the complicated mathematical equations. Hence, it is not an easy job to extend the formulation to more realistic wing structures such as stiffened multicell wings that are composed of the skins, stringers, ribs, and spars. In this report, a direct approach using the equilibrium equations for the composite sandwich plates proposed by Hwu and Hu 11,12 was employed to formulate the problems. The final mathematical formulation turns out to be a rather simple equation and can be proved to be equivalent to those derived in the literature. Due to its simplicity, the physical meaning of each equation is clear and hence is easy to be used to consider more complicated structures such as nonuniform stiffened composite multicell wings discussed in this report. Moreover, the important factors such as the warping restraint, transverse...
shear strain, shape of airfoil, aspect ratio, swept angle, fiber orientation, ply stacking sequence, stringers and spars, are all included in our formulation without adding too much complexity. The solving of the aeroelastic divergence is then performed by the equation written in an explicit matrix form.

2. Modelling of Stiffened Composite Multicell Wing Structures

The primary function of the wing structure is providing the lift for an aircraft, which is governed by the aerodynamic consideration. In addition to the aerodynamic pressure, there are other forces resisted by the wing structures such as the weight of the structures, fuels, engines, undercarriage system, and/or possible carried weapons, and the thrust of engines, etc. To sustain these loads, the wing structures are usually designed as that shown in Figure 1, which consists of axial members in stringers, bending members in spars, shear panels in the cover skin and spar webs, and planar members in ribs. If the cover skin of the wing is made of the composite laminates, the entire wing structure may be simulated by a composite sandwich plate in which the wing skins and stringers (including the spar flanges) are simulated as the faces while the spar webs and ribs are simulated as the core of the sandwiches. Since the wing cross section must have a streamline shape commonly referred to as an airfoil section, the thickness of the sandwich will not be a constant but a function of the airfoil. Moreover, as the usual sandwich assumptions, the thickness is not too small to neglect the transverse shear deformation. Based upon these considerations, a mathematical model for the composite sandwich plates proposed by Hwu and Hu 11 is applied in this report.

![Figure 1: Composite wing structure arrangements.](image1)

(i) Rigid Wing Chordwise Section

Due to the closely spaced stringers and the transverse stiffening members like wing spars and ribs, in aircraft analysis it is usually assumed that the wing chordwise section is rigid 13. Consistent with the chordwise-rigid postulation, the displacement fields may be assumed to be

\[ u = u_0 + z\beta_x, \quad v = v_0 + z\beta_y, \quad w = w_0, \]  

where \( u_0 = 0, \quad v_0 = v_0(y), \quad w_0 = w(y) - x\theta(y), \) \[ \beta_x = \theta(y), \quad \beta_y = \beta_j(y) + x\beta_j(y). \]  

\( w_j(y) \) denotes the deflection (positive upward) measured at the line of flexural center, which is now selected as the reference axis shown in Figure 2; \( \theta(y) \) is the twist around the flexural axis (positive nose up). The relation between \( \beta_x \) and \( \theta \) is due to the chordwise-rigid assumption which leads to \( \gamma_{xx} = 0 \). However, in spanwise direction the transverse shear deformation cannot be neglected for the thick plates. Hence, two extra functions \( \beta_f \) and \( \beta_r \) are needed for the representation of \( \beta_x \), where \( \beta_f \) denotes the rotation angle measured at the flexural axis and \( \beta_r \) stands for the rate of angle change in the \( x \)-direction.

![Figure 2: Geometry of the composite cantilever swept wings.](image2)

(ii) Modelling of Wing Skins, Stringers and Spar Flanges

In the wing structure design, most of the bending and axial loads are resisted by the stringers and spar flanges, while the wing skins resist almost all the in-plane shear forces. Hence, it is reasonable to model the wing skins, stringers, and spar flanges as the sandwich faces. If the stringers and spar flanges are considered to be the fibers of a pseudo-lamina, by the rule of mixture 14 the equivalent material properties of this pseudo-lamina may be written as
where $E, \nu, G$ and $A$ denote, respectively, the Young’s modulus, Poisson’s ratio, shear modulus and cross-section area. The subscript $L, T, s$ and $f$ denote the longitudinal direction, transverse direction, stringer and spar flange respectively. $A_p$ stands for the cross section area of pseudo-lamina. By adding this pseudo-lamina to the laminated composite skin, the face properties may be represented by the extensional, coupling and bending stiffnesses $A_y, B_y$ and $D_y$. Note that due to the shape of the airfoil, the location of each lamina $z_k$ is a function of $x$. If the upper and lower surfaces of the airfoil are represented by $f_u(x)$ and $f_l(x)$ (Figure 3), the stiffness matrices $A_y, B_y$ and $D_y$ of the wings are related to those of the corresponding laminated composite flat plate by

$$A_y(x) = A_y^u, \\
B_y(x) = B_y^u + f_u(x)A_y^u + f_l(x)A_y^l, \\
D_y(x) = D_y^u + 2f_u(x)B_y^u + 2f_l(x)B_y^l + f_u^2(x)A_y^u + f_l^2(x)A_y^l, \quad i, j = 1, 2, 6.$$  

Here, the superscript $F$ denotes the properties associated with the flat composite plates, while the superscripts $u$ and $l$ denote those of the upper and lower parts of the flat plate.

Due to the assumption given in (2), it is desirable to reduce the two-dimensional formulation given for the composite sandwich plates to an equivalent one-dimensional formulation. With this consideration, we like to integrate the stress resultants and bending moments with respect to $x$. Their relations with the mid-plane strains and curvatures may then be written as

$$\begin{bmatrix}
\tilde{N}_x \\
\tilde{M}_x
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{B}_{22} & \tilde{B}_{26} & \tilde{B}_{2*} & \tilde{B}_{26*} \\
\tilde{D}_{22} & \tilde{D}_{26} & \tilde{D}_{2*} & \tilde{D}_{26*}
\end{bmatrix}
\begin{bmatrix}
\nu_0' \\
\beta_0'
\end{bmatrix}.$$  

Here, the tilde ~ denotes integration with respect to $x$, the superscript * denotes multiplication by $x$, and the prime ' means differentiation with respect to $y$. For example,

$$\bar{D}_{23} = \int_{-c}^{c} D_{23} dx, \quad \bar{D}_{23*} = \int_{-c}^{c} D_{23} x dx,$$

$$\bar{D}_{23**} = \int_{-c}^{c} D_{23} x^2 dx, \quad v_0' = \frac{dv_x}{dy}, \ldots$$  

The lower and upper limits $-c_l$ and $c_l$ denote, respectively, the location of leading and trailing edges (Figure 2). The relations for the $N_x, N_y$ and $M_x$ are not shown since they play no roles in one-dimensional representation.

(iii) Modelling of Wing Spar Webs and Ribs

The main function of the wing spar webs and ribs is to resist the transverse shear force. Hence, it is suitable to model them as the sandwich cores. By assuming uniform transverse shear strain over the wing cross section, the equivalent transverse shear modulus $G_{yz}$ may be estimated by

$$G_{yz} = \tau_{yz} / \gamma_{yz}$$

where $\tau_{yz}$ is the average transverse shear stress and may be calculated by dividing the total transverse shear force $\bar{\tau}_0$ over the wing cross section $A_w$, i.e.,

$$\tau_{yz} = \bar{\tau}_0 / A_w.$$  

As to the transverse shear strains, it can be calculated by

$$\gamma_{yz} = \tau_{k} / G_{k} = \sum \tau_{k} A_{k} / \sum G_{k} A_{k} = \bar{\gamma}_{0} / \sum G_{k} A_{k}, \quad k = 1, \ldots, n_z.$$  

Here, $\tau_{k}, G_{k}$ and $A_{k}$ denotes the shear stress, shear modulus and section area of the $k$th spar web, and $n_z$ is the number of the wing spars. By this calculation, the equivalent transverse shear modulus $G_{yz}$ may be estimated as

$$G_{yz} = \sum_{k=1}^{n_z} G_k A_k / A_w.$$  

Figure 3: Sign convention of the composite sandwich plates.
As to the transverse shear modulus \( G_{xz} \), which shall be contributed by the wing ribs, no estimation is needed due to the assumption that \( \gamma_{xz} = 0 \). This also means that \( G_{xz} \) is assumed to be infinite under the construction of wing ribs. By proper integration, we now have
\[
\tilde{Q}_y = \tilde{A}_w(\beta_f + w_f^*) + \tilde{A}_w(\beta_f - \theta^*). \tag{9}
\]

(iv) Equilibrium Equations and Boundary Conditions

According to the postulation given in (2), the basic functions describing the deformation of the composite wing structures become \( v_0, w_f, \theta, \beta_f \) and \( \beta_r \). The equilibrium equations corresponding to these basic functions will then be obtained by integrating with respect to \( x \). The associated integration and the results are
\[
\begin{align*}
\int (\sum F_x = 0)dx : & \frac{d\tilde{N}_x}{dy} + \tilde{p}_y = 0, \\
\int (\sum F_z = 0)dx : & \frac{d\tilde{Q}_z}{dy} + \tilde{p} = 0, \\
\int (\sum M_y = 0)dx : & \frac{d\tilde{M}_y}{dy} + \tilde{m}_y = \tilde{Q}_y, \\
\int (\sum M_z = 0)dx : & -\tilde{M}_w + \frac{d\tilde{M}_w^*}{dy} + \tilde{m}_w^* = \tilde{Q}_w, \\
\int (\sum F_z = 0)dx : & \frac{d(\tilde{M}_w - \tilde{Q}_w^*)}{dy} + \tilde{m}_w - \tilde{p}_w^* = 0.
\end{align*}
\tag{10}
\]

During the integration, the boundary values of \( N_y, Q_z, M_y, \) and \( M_w \) have been assumed to be zero for the present one-dimensional modelling. In the above, please refer to (6) for the definition of tilde \( \sim \) and superscript \( \ast \). \( \tilde{N}_y, \tilde{Q}_z, \tilde{M}_w, \) and \( \tilde{M}_w^* \) are related to the basic functions \( v_0, w_f, \theta, \beta_f \) and \( \beta_r \) by those given in (5) and (9). \( \tilde{Q}_w^* \) and \( \tilde{M}_w^* \) may also be expressed in terms of the basic functions as
\[
\begin{align*}
\tilde{Q}_y^* &= \tilde{A}_w(\beta_f + w_f^*) + \tilde{A}_w(\beta_f - \theta^*), \\
\tilde{M}_w^* &= \tilde{B}_w^* v_0 + \tilde{D}_2^* \beta_f + \tilde{D}_6^* \beta_r + \tilde{D}_6^* (\theta^* + \beta_r).
\end{align*} \tag{11}
\]

The boundary conditions along \( y = \text{constant} \) can then be expressed as
\[
\begin{align*}
\tilde{N}_y &= \tilde{N}_y, \text{ or } v_0 = \tilde{v}_0, \\
\tilde{Q}_y &= \tilde{Q}_y, \text{ or } w_f = \tilde{w}_f,
\end{align*}
\]
\[
\tilde{M}_w - \tilde{Q}_w^* = \tilde{M}_w, \text{ or } \theta = \tilde{\theta}, \\
\tilde{M}_w = \tilde{M}_w, \text{ or } \beta_f = \tilde{\beta}_f, \\
\tilde{M}_w = \tilde{M}_w, \text{ or } \beta_r = \tilde{\beta}_r. \tag{12}
\]

By using the relations given in (5), (9) and (11), the equilibrium equations derived in (10) can be expressed in terms of five basic unknown functions \( v_0, w_f, \theta, \beta_f, \) and \( \beta_r \). Without considering estimation is needed due to the assumption that \( \gamma_{xz} = 0 \). This also means that \( G_{xz} \) is assumed to be infinite under the construction of wing ribs. By proper integration, we now have
\[
\tilde{Q}_y = \tilde{A}_w(\beta_f + w_f^*) + \tilde{A}_w(\beta_f - \theta^*). \tag{9}
\]

(v) Absence of Spanwise Loads

If the in-plane spanwise surface loads \( \tilde{p}_y \) can be neglected, the tenth-order system of equations may be reduced to an equivalent eighth-order system of four ordinary differential equations in terms of four basic unknown functions \( w_f, \theta, \beta_f, \) and \( \beta_r \). With \( \tilde{p}_y = 0 \), the equilibrium equation in \( y \)-direction (10) leads to \( \tilde{N}_y = \text{constant} \). If no in-plane spanwise loads are applied at wingtip, this constant value will then be identical to zero. That is, \( \tilde{N}_y = 0 \) along the entire wing. Substituting this result into (5) \(_1\), \( v_0 \) may be expressed in terms of \( \beta_f, \beta_r, \) and \( \theta + \beta_r \). Thus, the relations given in (5) and (11) \_2 can be simplified as
\[
\begin{align*}
\tilde{M}_w &= \left[ \begin{array}{c}
\tilde{D}_{22} \\
\tilde{D}_{26} \\
\tilde{D}_{26} \\
\tilde{D}_{26}
\end{array} \right] \left[ \begin{array}{c}
\beta_f' \\
\beta_r' \\
\theta + \beta_r
\end{array} \right], \quad \text{(13)}
\end{align*}
\]
where
\[
\begin{align*}
\tilde{D}_j &= \tilde{D}_j - \tilde{B}_2^j \tilde{B}_2^j, \\
\tilde{D}_j^* &= \tilde{D}_j - \tilde{B}_2^j \tilde{B}_2^j, \\
\tilde{D}_j^* &= \tilde{D}_j - \tilde{B}_2^j \tilde{B}_2^j, \\
\tilde{D}_j^* &= \tilde{D}_j - \tilde{B}_2^j \tilde{B}_2^j, \quad i, j = 2, 6.
\end{align*} \tag{14}
\]

Using the simplified relations derived in (13) together with (9) and (11), the remained four equilibrium equations (10) \_2 - (10) \_5 then constitute an eighth-order system of four ordinary differential equations in terms of four basic unknown functions \( w_f, \theta, \beta_f, \) and \( \beta_r \). Without considering the first condition of equation (12), the corresponding boundary conditions listed in (12)
also reduce to four conditions per end.

Therefore, for a composite wing structure with the spanwise loads as well as the distributed moments neglected (i.e., \( \tilde{\rho}_p = \tilde{N}_p = 0 \) and \( \tilde{m}_p = \tilde{\nu}_p = 0 \)), the equilibrium equations and boundary conditions given in (10) and (12) can be reduced to

\[
\frac{d\tilde{Q}_y}{dy} = -\tilde{\rho}, \quad \frac{d(M_{\nu} - \tilde{Q}_y^*)}{dy} = \tilde{p}^*, \tag{15}
\]

and

\[
w_f = \theta = \beta_f = 0, \quad \text{at} \quad y = 0 \quad \text{(wing root)},
\]

\[
\tilde{Q}_y = M_{\nu} - \tilde{Q}_y^* = M_{\nu} = \tilde{M}_{\nu}^* = 0, \quad \text{at} \quad y = l \quad \text{(wing tip)}.
\]

(vi) **Warping Restraint Effects**

The importance of the warping restraint effects has been discussed vastly in Librescu’s series work [7-10]. Due to its importance, the warping restraint effects will not be neglected in this report. This effect has been considered when we include \( \beta_f^* \) in the expression of \( \varepsilon_f \). In other words, the free warping assumption can easily be done by letting \( \beta_f^* = 0 \) in (5), (11) and all the following related equations.

(vii) **Governing Equations**

To solve the problem established by the equilibrium equations (15) and the boundary conditions (16), we first express all the functions in terms of four basic unknown functions \( w_f, \theta, \beta_f, \beta_f' \). From (9), (11), and (13), the relations between \((\tilde{Q}_y, M_{\nu}, \tilde{Q}_y^*, \tilde{M}_{\nu}^*)\) and \((w_f, \theta, \beta_f, \beta_f')\) can be written by matrix notation as

\[
F = K_0\Lambda + K_1\Lambda', \tag{17a}
\]

where

\[
F = \begin{bmatrix} \tilde{Q}_y \\ M_{\nu} \\ \tilde{Q}_y^* \\ \tilde{M}_{\nu}^* \end{bmatrix}, \quad \Lambda = \begin{bmatrix} w_f \\ \theta \\ \beta_f \\ \beta_f' \end{bmatrix}, \quad K_0 = \begin{bmatrix} 0 & 0 & \tilde{A}_{44} & \tilde{A}_{44} \\ 0 & 0 & \tilde{D}_{26} & \tilde{D}_{26} \end{bmatrix}, \quad K_1 = \begin{bmatrix} \tilde{A}_{44} - \tilde{A}_{44}^* & 0 & 0 \\ 0 & \tilde{D}_{26} & \tilde{D}_{26}^* & \tilde{D}_{26}^* \end{bmatrix}. \tag{17b}
\]

The system of equations (15) with boundary conditions given in (16) can then be solved by using the technique of Laplace integral transform. This technique transforms (15) to a linear algebraic system of equations as

\[
\begin{align*}
\tilde{Q}_y - \tilde{Q}_y(0) + \tilde{p} = 0, \\
\tilde{M}_{\nu}^* - \tilde{M}_{\nu}(0) + \tilde{\nu}_y(0) - \tilde{p}^* = 0, \\
\tilde{M}_{\nu}^* - \tilde{M}_{\nu}(0) - \tilde{Q}_y = 0,
\end{align*}
\]

or in matrix notation as

\[
\mathbf{S}\hat{\mathbf{F}}(s) = \mathbf{J}\hat{\mathbf{F}}(0) + \mathbf{P}(s), \tag{18b}
\]

where

\[
\mathbf{S} = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & -s & 0 \\ -1 & s & 0 & 0 \\ 0 & -1 & -1 & s \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{P}(s) = \begin{bmatrix} \tilde{p}(s) \\ 0 \\ 0 \end{bmatrix}.
\]

The overhat \( \hat{\cdot} \) denotes the Laplace transform, e.g., \( \hat{\mathbf{F}}(s) = L[\mathbf{F}(y)] \). By (17a), we have

\[
\mathbf{F}(0) = \mathbf{K}_0\Lambda(0) + \mathbf{K}_1\Lambda'(0).
\]

The boundary condition (16) means that \( \Lambda(0) = 0 \). Hence, \( \mathbf{F}(0) = \mathbf{K}_0\Lambda(0) \). The Laplace transform of (17a) plus the boundary condition, \( \Lambda(0) = 0 \), leads to

\[
\mathbf{S}(\mathbf{K}_0 + s\mathbf{K}_1)\hat{\mathbf{A}}(s) = \mathbf{J}\hat{\mathbf{A}}(0) + \mathbf{P}(s). \tag{19}
\]

Substituting these results into (18b), we get the governing equations for the composite wing structures in the transformed domain of \( w_f, \theta, \beta_f, \beta_f' \), as

\[
\mathbf{S}(\mathbf{K}_0 + s\mathbf{K}_1)\hat{\mathbf{A}}(s) = \mathbf{J}\hat{\mathbf{A}}(0) + \mathbf{P}(s),
\]

and the boundary conditions remained to be satisfied are those given in (16). By using the matrix notation and the relation given in (17a), (16) can be rewritten as

\[
\mathbf{J}\hat{\mathbf{F}}(l) = \mathbf{J}[\mathbf{K}_0\hat{\mathbf{A}}(l) + \mathbf{K}_1\hat{\mathbf{A}}'(l)] = 0. \tag{20}
\]

In order to check the modeling described in the last section, several numerical examples are now undergoing.

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References