Indentation by multiple rigid punches on two-dimensional anisotropic elastic or viscoelastic solids

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ABSTRACT

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A two-dimensional anisotropic elastic or viscoelastic solid indented by multiple rigid punches is considered. By taking the rigid body translation and rotation of each punch as the additional variables to the usual contact boundary element method (BEM), we propose a brand-new BEM in which the additional equations come from the force/moment equilibrium of each punch. In this newly developed BEM, the selected fundamental solutions are valid for the general anisotropic elastic/viscoelastic solids with or without holes/cracks. Under this consideration, no meshes are required on the boundaries of rigid punches and holes/cracks, which makes it much more accurate and efficient than the conventional contact BEM. This method is valid for the frictionless or frictional contact surface, and the punches can be in equilibrium status or in quasistatic sliding condition. If the solids are anisotropic elastic, it is applicable for both incomplete and complete indentation. Since the extension from elastic to viscoelastic is based upon the elastic-viscoelastic correspondence principle that requires all the boundaries be time-independent, our method is valid only for complete indentation if the indented solid is viscoelastic. In addition to this new BEM, in order to verify our results for the cases with viscoelastic solids, by proper use of Laplace transform and its inversion, new analytical solutions for the indentation by a flat-ended/parabolic punch are also presented for the half-plane made by anisotropic viscoelastic materials.

1. Introduction

In practical engineering, several different problems can be idealized and simulated as an anisotropic elastic/viscoelastic solid indented by multiple rigid punches, such as rough surfaces [1–3], tyre/road contact [4, 5], metal forming [6], and foundation/soil interaction [7]. However, due to the complexity raised by the directional dependence of anisotropy, time dependence of viscoelasticity, non-linearity of contact, frictional surface, and interaction of multiple punches, very few studies involving all these subjects together can be found in the literature. Most of the researches focus on one or two subjects such as anisotropic elastic half-plane indented by one or two rigid punches [8–12], contact of two elastic bodies [13–22], one rigid punch on an isotropic elastic/viscoelastic half-plane [23–25], a periodic system of indenters (rough surface) on an isotropic elastic half-space [26–28], and one rigid indenter on an orthotropic viscoelastic half-space [29, 30]. Their contact condition is either sliding [12, 13, 28], frictionless or frictional [8–10, 14–27, 29–30], or bonded [11, 13]. In these studies some idealized problems can be solved analytically [8–13, 23–30], while most of the complicated problems reply on the numerical methods such as finite element method (FEM) [14–18] or boundary element method (BEM) [15, 19–22]. Since contact region is usually a small portion of the boundaries, BEM which discretizes only the boundaries of the entire body is deemed to be a suitable method for contact problems [19–22]. Furthermore, by taking advantage of the fundamental solutions required in BEM, we may select some special and available fundamental solutions to solve the special problem such as the one with holes, cracks, or inclusions [22]. With the special fundamental solution, no meshes are required on the specific locations such as holes, cracks, or inclusions, which is really beneficial since these locations usually need very fine meshes on FEM or BEM. If all the punches are considered to be rigid, the movement of punches can be represented by rigid body translation and rotation. And again, no meshes are required on the rigid punches if proper mathematical simulation can be proposed.

With the above consideration, in this paper by taking the rigid body translation and rotation of each punch as the additional unknown variables to the usual contact BEM we propose a brand new BEM for solving the contact problems with multiple rigid punches on anisotropic elastic solids. Here, the profile of each punch head can be arbitrary and different, the contact surface can be frictionless or frictional, the punches can be in equilibrium or sliding, the number of rigid punches is not restricted, the elastic body can be any kind of anisotropic including orthotropic or isotropic, and the indentation can be incomplete or complete. Based on the proposed contact BEM and the elastic-viscoelastic

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