Boundary element method for two-dimensional frictional contact problems of anisotropic elastic solids

Van Thuong Nguyen, Chyanbin Hwu*

Department of Aeronautics and Astronautics, National Cheng Kung University, Tainan, Taiwan, ROC

A R T I C L E   I N F O

Keywords:
Boundary element method
Frictional contact
Anisotropic elastic
Contact constraint
Incremental load

A B S T R A C T

The conventional boundary element method solving for the problems of two-dimensional anisotropic elastic solids with prescribed traction and/or prescribed displacement boundary conditions is extended to the frictional contact problems. A complete system of linear equations is constructed by boundary integral equations and contact constraint relations. The contact solutions are obtained by using an efficient, iterative and fully incremental loading technique. By using this technique, the nonlinearity raised by unknown contact region and unknown slip direction of frictional contact can be approximated by the accumulation of linear increments. The incremental load is determined by using the load extrapolation technique that allows only one or two node pairs come into contact in each iteration. The slip direction of frictional contact is decided by referring to the relative tangential slip in the frictionless state. To avoid reassembling the whole system equations in each iteration, a suitable arrangement of the equation system is made and a fast solver is adopted to get the solution without resolving the entire system of equations. When the contact bodies contain holes, cracks or inclusions, we use a special boundary element whose fundamental solution satisfies the boundary condition along the hole/crack/inclusion boundary. The validation of the proposed method is demonstrated through several numerical examples, which further lead to the discussion of the effects of friction coefficient, material anisotropy, holes, cracks and inclusions on contact.

1. Introduction

Contact problems of anisotropic elastic solids are challenging problems since the complexities arise not only from the involving of directional dependence of materials but also from the non-linearity caused by the unknown contact area and frictional contact. The problems are even more difficult if the defects such as holes, cracks, and/or inclusions exist inside the contact bodies. In the last few decades, many researchers have put great efforts on handling contact problems either by using analytical or numerical treatments. By using the analytical or semi analytical treatment, several works can be found such as [1–3] for isotropic elastic solids, [4,5] for transversely isotropic elastic solids, and [6–10] for anisotropic elastic solids. Even some of these studies worked on anisotropic elastic solids, their results are applicable for idealized problems such as contact of rigid punches on a half plane without holes, cracks, or inclusions due to the limitation of analytical solutions.

Among the possible numerical approaches that can be used to handle the contact problems, finite element method (FEM) and boundary element method (BEM) are two popular numerical methods. References of using FEM to solve anisotropic elastic contact problems can be found in [11–15]. A common issue of these works is that fine mesh in contact regions is usually required in order to obtain accurate results. BEM has shown its advantages over the other methods since it reduces the problems by one dimension and the boundaries are the primary concern in contact problems [16–22]. Although BEM was applied to the contact problems long time ago, most of the studies are limited to the cases with isotropic elastic solids. Due to the complexity of the associated fundamental solutions, relatively few studies focus on the contact of anisotropic elastic solids [23–25].

In this work, the conventional BEM solving for the problems of anisotropic elastic solids with prescribed-traction and/or prescribed-displacement boundary conditions is extended to the problems with frictional contact conditions. The contact solutions are obtained by using an efficient, iterative and fully load-incremental technique proposed in [18]. This method has the advantage that the contact constraints can be directly and exactly enforced at the contact node pairs. In addition, by suitable arrangement of the structural matrix, we only need to update the parts of the final system matrix related to the contact conditions instead of reassembling the whole system equations during the calculation process. Furthermore, the fast solver based on Sherman-Morrison formula [26] is used to update the solution from one iteration to the next without resolving the final system of equations.

To demonstrate the correctness and applicability of the present approach, several examples such as a flat-ended or parabolic punch on an