Three-dimensional boundary element analysis for anisotropic elastic solids and its extension to piezoelectric and magnetoelectroelastic solids

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ABSTRACT

This article presents three-dimensional (3D) analysis of generally anisotropic elastic, piezoelectric, piezomagnetic, and magnetoelectro-elastic solids by boundary element method (BEM). The associated Green's functions of displacements and tractions in both complex and real forms together with their first derivatives are derived completely in the first time by using Radon–Stroh formalism. Like the Stroh formalism for two-dimensional problems, the solutions for the solids with these different material types all bear exactly the same mathematical forms distinguished by the contents and dimensions of the related matrices and vectors. This feature provides a big advantage for computer programming. To emphasize the development of 3D-BEM, the system of algebraic equations to calculate the nodal displacements and nodal tractions along the boundary of 3D solids is formulated. The relations for calculating strains and stresses at the boundary nodes as well as at the interior points are also derived. After the employment of Green's functions to the newly formulated 3D-BEM, we found that the 3D-BEM with complex form solution is computationally more efficient than that with real form solution, although the opposite performance was observed for a single point calculation of Green's function. To verify the correctness of our solutions, numerical examples of a cube with or without through hole are presented as illustrations of our successful implementation in the BEM.

1. Introduction

For boundary element method (BEM), evaluation of Green's functions plays a key role in computing its associated boundary integrals. Green's functions for both two-dimensional (2D) and three-dimensional (3D) isotropic elastic bodies have been well documented in the literature [1–14]. For 2D anisotropic elastic solids, classical solutions can be found in [1–3]. Although several different forms of Green's functions for 3D anisotropic elastic solids have been proposed [4–14] and a well-comprehensive review of Green's functions in anisotropic media can be found in the book of Pan and Chen [9], most of the studies still end at the discussions of Green's function not the 3D-BEM itself. In a recently published paper [10], several different expressions of Green's function and its derivatives such as an integral on the oblique plane [4,5], an integral on the vertical or horizontal plane [6,7], an analytical solution expressed in terms of Stroh's eigenvalues [11,12] or eigenvectors [13,14], and an approximated series expansion [8], have been investigated and compared. They conclude that the real form solution obtained by 2D Radon–Stroh formalism [10], which is expressed in terms of an integral on the vertical or horizontal plane, performs with the highest efficiency and accuracy. However, no further publication can be found for its practical application to 3D-BEM. Similarly, many discussions can be found for the generalization of Green's functions from anisotropic elastic solids to the general anisotropic piezoelectric, piezomagnetic, and magnetoelectroelastic (MEE) solids [9,15–22], but still not for the generalization of 3D-BEM. By the detailed investigation shown later in this paper, opposite performance has been observed for 3D-BEM. In other words, 3D-BEM with complex form Green's function is more efficient than that with real form.

The above-mentioned Radon–Stroh formalism is an approach based on Stroh's complex variable formalism of two-dimensional anisotropic elasticity by using the Radon transform to map a 3D solid to a 2D plane [6,7,23]. The well-known Stroh formalism can be used to obtain solutions for a particular 2D problem, by which the corresponding 3D solutions can be constructed simply by integrating those of the 2D cases such as full space, half space and bi-materials [3,6,9]. For saving computational time in calculating Stroh's eigenvalues and eigenvectors for the complex form solutions, real form solutions can be formulated using Stroh's identities as presented in [3,10]. Although it looks well to get 3D solutions by Radon–Stroh formalism, almost all Green's function presented in the literature are expressed for the displacements and their first and second derivatives without directly showing another important expressions for tractions. Since displacements and tractions are two equally important functions in BEM, the simplicity of their solution form will influence the efficiency of numerical calculation. In order to provide a simple form of Green's functions, in this paper the Radon–Stroh formalism is re-derived, and the solutions for displacements and

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