Hygrothermal Stresses in Unsymmetric Laminates Disturbed by Elliptical Holes

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1 Introduction

Composite laminates are increasingly being used not only in traditional areas like aerospace, but also in many engineering applications. Some of these applications are the structures under hygrothermal environment. Although the hygrothermal effects on holes in laminates have been widely discussed, due to mathematical infeasibility most of the analytical solutions found in the literature are for two-dimensional problems or for mechanical loading conditions, for isotropic materials, or for special laminates [1–3], not for the general composite laminates under hygrothermal environment. For a unidirectional laminate the coefficients of thermal and moisture expansion, like its other properties, change with direction. Thus, the hygrothermal changes result in unequal strains in the longitudinal and transverse directions. Hygrothermal strains do not produce a resultant force or moment when the body is completely free to expand, bend, and twist. However, for a composite laminate each individual lamina is not completely free to deform. The lamina stresses are therefore induced by the constraints placed on its deformation by adjacent lamina [4,5]. Like the cases of mechanical loading, the existence of holes in laminates will cause high stress concentration around holes under hygrothermal environment. Moreover, the unsymmetry of laminates will cause coupling between stretching and bending, which may complicate the analysis. Due to the designable characteristics of composite laminates, sometimes the engineering designers want to utilize the coupling effects to do something that cannot be achieved by using metallic or symmetric laminates. Thus, the study of hygrothermal stress analysis in unsymmetric laminates becomes important for practical engineering design.

Recently, we developed a Stroh-like formalism for coupled stretching-bending analysis of composite laminates [6] by extending the Stroh formalism for two-dimensional linear anisotropic elasticity [7,8] and successfully solved the hole problems in general composite laminates [9]. It is expected that with the help of our Stroh-like formalism, we may easily solve the corresponding hygrothermal problems of holes in general composite laminates. Like the extension of Stroh formalism to anisotropic thermoelasticity [3], we extend our Stroh-like formalism to the hygrothermal stress analysis of laminates in this paper. By using this formalism, the general solutions for hygrothermal stresses in unsymmetric laminates disturbed by an elliptical hole subjected to uniform heat flow and moisture transfer in the x_1-x_2 plane and x_3 direction are now obtained analytically. To illustrate our exact solutions, three numerical examples discussing the above hygrothermal effects of the holes in unsymmetric laminates are presented in this paper.

The ANSYS finite element software package is also used to compare both numerical examples, which shows our present solutions are simple and correct.

2 Basic Equations

In a fixed rectangular coordinate system x_i, i=1,2,3, let U_i, \sigma_{ij}, e_{ij}, T, H, q_i, and m_i be, respectively, displacement, stress, strain, change in temperature, change in moisture content, heat flux, and moisture transfer. If the coupling terms between the elastic deformation, heat conduction, and moisture transport are neglected, the heat conduction, the moisture diffusion, the strain-displacement relation, the constitutive law, the force, heat and moisture equilibrium equations for linear anisotropic elastic materials under static loading, and small deformation conditions can be written as [10]

\[ q_i = -k_i^t T_i \], \quad m_i = -k_i^h H_i \], \quad e_{ij} = \frac{1}{2}(U_{ij,j} + U_{ij,i}) \],

\[ \sigma_{ij} = C_{ijkl} e_{kl} - C_{ijkl} a_k^t T - C_{ijkl} a_k^h H \].

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