STROH FORMALISM AND ITS EXTENSIONS TO COUPLED INPLANE-BENDING PROBLEMS

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ABSTRACT

Stroh formalism for two-dimensional linear anisotropic elasticity has been discussed vastly in the literature. Its extension to the coupled inplane-bending problems was just emerging recently, and was called Stroh-like formalism. In order to transfer all the related formulae and mathematical techniques of Stroh formalism to Stroh-like formalism, an exact comparison is necessary. To have the exact comparison, same notations are used in these two formalisms if they bear the same physical meaning. Hence, some of the expressions published recently in the previous works have been rewritten and compared in this paper. Moreover, the uncoupling of inplane and antiplane problems, and the uncoupling of inplane and bending problems are also discussed and compared. From the comparison and discussion given in this paper, we know that further detailed derivation for the unsolved problems may be avoided if their corresponding problems have been solved previously.

1. INTRODUCTION

For two-dimensional linear anisotropic elasticity, there are two major complex variable formalisms in the literature. One is Lekhnitskii formalism [1,2] which starts with the equilibrated stress functions followed by constitutive laws, strain-displacement relations and compatibility equations; the other is Stroh formalism [3,4] which starts with the compatible displacements followed by strain-displacement relations, constitutive laws and equilibrium equations. Due to the special feature that Stroh formalism possesses an eigen-relation which relates the eigen-modes of stress functions and displacements to the material properties, Stroh formalism becomes more attractive than Lekhnitskii formalism, especially when the book (Ting, [5]) emphasized on Stroh formalism was published.

Due to the success of Stroh formalism for two-dimensional problems, recently some of the researchers tried to extend Stroh formalism to the plate bending problems or even to the coupled stretching-bending problems [6–9]. This kind of problems occurs frequently in the classical lamination theory (Jones, [10]), but due to the mathematical difficulties only very few of them have been solved analytically. For example, even the very common engineering problems like the hole/crack/inclusion problems were just solved recently by using the newly developed Stroh-like formalism [11,12].

Due to the resemblance of Stroh formalism and Stroh-like formalism, their solutions for two-dimensional problems and coupled stretching-bending problems are really very alike. Therefore, with the help of Stroh formalism, one may solve the coupled stretching-bending problem easily by using Stroh-like formalism. In this paper, we like to go further to provide the exact comparison of these two seemingly alike formalisms. To have the exact comparison, same notations will be used in these two formalisms if they bear the same physical meaning. Hence, some appearances of Stroh-like formalism published in our previous works are changed. By this detailed comparison, it is now very possible that we can get the solutions for the coupled stretching-bending problems or plate bending problems directly from the comparison with their corresponding two-dimensional problems.

2. STROH FORMALISM FOR COUPLED INPLANE AND ANTIPLANE PROBLEMS

In a fixed rectangular coordinate system \( x_i, i = 1, 2, 3 \), let \( u_i, \sigma_{ij}, \varepsilon_{ij} \) be, respectively, the displacement, stress and strain. The strain-displacement equations, the stress-strain laws, and the equations of equilibrium for anisotropic elasticity are