A Key Matrix N for the Stress Singularity of the Anisotropic Elastic Composite Wedges*

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By employing the Stroh formalism, a general solution satisfying the basic laws of two-dimensional linear anisotropic elasticity has been written in a complex variable formulation. To study the stress singularity, suitable stress functions have been assumed in the exponential form. The singular order near the anisotropic elastic composite wedge apex can then be found by satisfying the boundary conditions. Since there are many material constants and boundary conditions involved, the characteristic equation for the singular order usually becomes cumbersome or leaves in the form of a system of simultaneous algebraic equations. It is therefore difficult to get any important parameters to study the failure initiation of the composite wedges. Through a careful mathematical manipulation, a key matrix \( N \) that contains the information of material properties and wedge geometries has been found to be a dominant matrix for the determination of the singular order. A closed-form solution for the order of stress singularity is thus written in a simple form. Special cases such as the wedge corners, cracks, interfacial joints or cracks, a crack terminating at the interface, etc. can all be studied in a unified manner.

Key Words: Anisotropy, Elasticity, Composite Material, Stress Singularity, Wedge

1. Introduction

The stress singularity generally occurs at the location of discontinuity. The discontinuity may come from geometries, materials or loads, of which the typical examples are, respectively, cracks, multi-player media, or point forces. Due to the extremely high stresses near the points of discontinuity, failure is usually initiated at such locations. The study of the singularity is generally helpful for the understanding of failure initiation. The nature of stress singularity for the above typical examples has been investigated by many researchers and is illustrated in standard texts such as Anderson[1], Jones[2], and Johnson[3]. Geometrical discontinuity other than cracks, which has also been studied vastly is the singular stresses at wedge corners. They have been studied for single wedges by Williams[4], England[5], Bogey[6], Stern and Soni[7], Chen[8], etc., and for bonded wedges by Bogey[9,10], Dundur[11], Hein and Erdoga[12], Theocari[13], Lin and Hartman[14], Reed[15,16], Chen and Nishitani[17], Ding, et al.[18], Ting[19], Berger, et al.[20], Chen[21] and Desmorat and Leckie[22], etc. However, due to the mathematical difficulties, most of the results leave the solutions to a system of simultaneous algebraic equations. Only Ting[23] provides the explicit closed-form solutions by the introduction of a transfer matrix for each wedge.

Through the eigen-relation of Stroh formalism for two-dimensional problems (Stroh[24], Ting[25]), a key matrix \( \mathbf{N} \) related to the fundamental elasticity matrix \( \mathbf{N} \) is introduced in this paper. By rigorous mathematical proof, one can see that the transfer matrix introduced by Ting[26] is a combination of this key matrix. From the present study, we find that this matrix includes the information of material properties and wedge angles, and each wedge can be represented by its own \( \mathbf{N} \). Therefore, no matter how many wedges are bonded together, the closed-form solutions for the order of stress singularity can easily be constructed by simple multiplication of \( \mathbf{N} \) for bonded wedges.