EXPLICIT EXPRESSIONS OF THE FUNDAMENTAL ELASTICITY MATRICES OF STROH-LIKE FORMALISM FOR SYMMETRIC/UNSYMMETRIC LAMINATES

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ABSTRACT

Based upon our recent development of Stroh-like formalism for symmetric/unsymmetric laminates, most of the relations for bending problems can be organized into the forms of Stroh formalism for two-dimensional problems. Through the use of Stroh-like formalism, the fundamental elasticity matrices \( N, S, H \) and \( L \) appear frequently in the real form solutions of plate bending problems. Therefore, the determination of these matrices becomes important in the analysis of plate bending problems. In this paper, by following the approach for two-dimensional problems, we obtain the explicit expressions of the fundamental elasticity matrices for symmetric and unsymmetric laminates, which are all expressed in terms of the extensional, bending and coupling stiffnesses of the composite laminates.

Keywords: Composite laminates, Anisotropic elasticity, Plate bending problems, Stroh formalism, Fundamental elasticity matrices.

1. INTRODUCTION

The Stroh formalism [1,2] has been shown to be a powerful tool for the study of two-dimensional deformation of anisotropic elastic materials. The details and applications can be found in Tung’s book [3]. Due to the special feature that Stroh formalism possesses an eigen-relation which relates the eigen-modes of stress functions and displacements to material properties, the resulting real form solutions can usually be expressed in terms of the fundamental elasticity matrices. Therefore, the explicit expressions of the fundamental matrices have been discussed vastly for the two-dimensional problems [3].

The extension of Stroh formalism to the analysis of bending of anisotropic elastic plates has been introduced by Lu and Mahrenholtz [4] and corrected by Cheng and Reddy [5]. However, their final eigen-relation is not completely similar to that of the Stroh formalism for two-dimensional problems. Therefore, it is not easy to apply the same method developed for the two-dimensional problems to the bending problems. Basically, all these formalisms are developed with the displacement functions as the basis functions. Recently, by using partially displacements and partially stress functions as the basis functions, Hwu [6,7] developed a Stroh-like formalism for the bending theory of anisotropic plates, which can also be directly applied to the symmetric and unsymmetric composite laminates. The advantage of this Stroh-like formalism is that it is really very similar to the Stroh formalism for two-dimensional problems. By using this formalism, we successfully obtained the analytical solutions for the problems of anisotropic plates with holes/cracks/inclusions subjected to out-of-plane bending moments [8]. The resulting real form solutions are all written in terms of the fundamental elasticity matrices. With this advantage, in this paper we like to find the explicit expressions of the fundamental elasticity matrices.

2. STROH-LIKE FORMALISM FOR SYMMETRIC/UNSYMMETRIC LAMINATES

2.1 Unsymmetric Laminates

By combining the assumptions for lamination theory, the kinematic relations, the constitutive laws and the equilibrium equations, the governing equations for the composite laminates can be written in terms of three unknown mid-plane displacement functions \( u_0, v_0 \) and \( w \) as [9]