Electromechanical analysis of defects in piezoelectric materials

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Abstract. In this paper, electromechanical analysis is performed based on an extended Stroh formalism which considers the constitutive law of a linear anisotropic piezoelectric medium, the kinematics of two-dimensional small deformations and static equilibrium. By this formalism, several analytical closed-form solutions are obtained for holes, cracks or interface cracks in an infinite piezoelectric medium under various loading conditions. To have a practical use for engineering analyses, the Green’s functions obtained from point load condition are embedded into the usual boundary integral equations. Thus, a special boundary element is developed to deal with the problems with finite boundaries.

1. Introduction

Due to the rapid development of intelligent space structure and mechanical systems, advanced structures with integrated self-monitoring and control capabilities are increasingly becoming important. It is also well known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. Due to this intrinsic coupling phenomenon, piezoelectric materials are widely used as sensors and actuators in intelligent advanced structure design. When subjected to mechanical and electric stresses in service, these piezoelectric materials can fail due to defects such as cracks, holes, etc. arising during their manufacture. Therefore, it is interesting to study the electromechanical behavior of piezoelectric materials with defects.

There have been relatively few research efforts in electromechanical modeling of cracks or holes in piezoelectric materials. Early in 1975, Barnett and Lothe [1] extended Stroh’s six-dimensional (6D) framework [2] to an eight-dimensional (8D) formalism to treat dislocations and line charges in anisotropic piezoelectric insulators. At that time, more researchers paid attention to fracture problems in piezoelectric media, such as closed-form solution of the antiplane fracture problem [3], (3D) eigenfunction analysis [4] and plane strain piezoelectric problem by means of complex variables theory [5]. They concluded that for certain ratios of applied electrical load to mechanical load, crack arrestment can be observed and the electric fields generally tend to slow crack growth. Suo et al [6] studied cracks either in piezoelectrics, or on interfaces between piezoelectrics and other materials such as metal electrodes or polymer matrices. They discovered a new type of singularity around interface crack tips and solved a class of boundary value problem involving many cracks on the interface between half spaces. Kuo and Barnett [7] obtained analytical expressions for the crack tip singularities for the problems of a crack in a homogeneous piezoelectric medium and of an interfacial crack between two dissimilar piezoelectric media for a variety of crack surface boundary conditions.

In this paper, several new results for cracks or holes in piezoelectric materials are presented by applying the extended Stroh formalism for anisotropic piezoelectric elasticity. These results are exact closed-form solutions for infinite domain problems. To have a practical use for engineering analyses, the Green’s function obtained from point load condition is embedded into the usual boundary integral equations. Thus, a special boundary element is developed in the present paper to deal with the problem of finite piezoelectric plates containing holes or cracks.

2. Electromechanical analysis

For an anisotropic linear piezoelectric medium, the constitutive law may be written as

\[
\sigma_{ij} = C_{ijkl} u_{k,l} - e_{kij} E_k
\]

\[
D_i = e_{ikj} u_{j,l} + \epsilon_{ik} E_k
\]

(1)

where \(E_i\) is the electrical field with unit N/C or V/m; \(D_i\) is the electrical displacement with unit \(\text{Cm}^{-2}\) or \(\text{Nm}^{-1}\ \text{V}^{-1}\); \(e_{kij}\) is the piezoelectric stress tensor with unit \(\text{Cm}^{-2}\) or \(\text{Nm}^{-1}\ \text{V}^{-1}\) and \(\epsilon_{ik}\) is the permittivity tensor with unit \(\text{C}^2\ \text{m}^{-2}\) or \(\text{N} \text{V}^{-2}\). Besides the full symmetry conditions of the elasticity tensor \(C_{ijkl}\), the piezoelectric stress tensor \(e_{kij}\) and the permittivity tensor \(\epsilon_{ik}\) also satisfy the full symmetry conditions. The governing equations for the