Interactions Between Dislocations and Anisotropic Elastic Elliptical Inclusions

1 Introduction

Interactions between dislocations and inclusions have been a topic of considerable research. Greater understanding of frictional defects can be gained through the solution of suitable elasticity problems. The solutions of dislocations are frequently used as a kernel function of an integral equation to consider the interactions between dislocations and cracks (Zhodan et al., 1974; Patton and Sanare, 1990). The problem of a circular elastic inclusion near an edge dislocation was solved in terms of Airy's stress potentials by Dundurs and Nura (1964). Later, Dundurs and Stroevsky (1965) solved the rotated problem where the dislocation is within the circular inclusion. Recently, the analytic solutions of isotropic elliptic inhomogeneity were obtained by Stagni and Lütz (1983) for a dislocation located outside an elliptic inhomogeneity and by Warren (1983) for a dislocation inside an elastic elliptic inhomogeneity. A closed-form solution is obtained by Sastri and Kurr (1986) for a dislocation near a rigid elliptical inclusion, in which particular attention is paid to the rigid-body rotation of the inclusion relative to the dislocation.

While these solutions are generally in isotropic materials, the concept of an elastic, isotropic inclusion is an idealization. All real crystals should be considered to be anisotropic. Anisotropic theory may lead to useful results, but for some cases it is an inadequate approximation. Moreover, there are increasing numbers of observations of sufficient accuracy to warrant comparison with more precise anisotropic calculations. Also, some effects, such as the instability of some straight dislocations with respect to break up into zigzag shape, require anisotropic theory even for their qualitative explanation (Hirth and Lothe, 1982). Hence, it is necessary to analyze such interaction problems by using anisotropic theory. In this study, the Stroevsky's formalism (1985) for anisotropic elasticity combined with the method of analytical approximations (Muskhelishvili, 1954) is utilized to solve the general analytical solution for the interactions between dislocations and anisotropic elastic elliptical inclusions.

2 Anisotropic Elasticity

The stress-strain relation obeying Hooke's law for the generally anisotropic material is usually expressed as

\[ \sigma = C_{ijkl} \varepsilon_{ij} \]

where \( \sigma \) and \( \varepsilon \) denote, respectively, the stresses and strains. \( C_{ijkl} \) are the elastic constants which are assumed to be fully symmetric and positive definite. For a two-dimensional problem, it has been shown (Eshelby, 1952; Shah et al., 1959; Lekhnitskii, 1963; Ting, 1980) that the elastic field can be represented in terms of three complex functions, \( f(z_1), f(z_2), f(z_3) \), each of which is homogeneous in its argument, \( z = \alpha \zeta + p_0(\zeta) \). Here, \( p_0, \alpha = 1, 2, 3 \), are the material eigenvalues which are complex numbers with positive imaginary part and can be solved as roots of a sixth-order polynomial (Eshelby et al., 1953). With these holomorphic functions, the displacements \( u \), stress functions \( \phi \), and stress \( \sigma_0 \) can be represented as

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