FRACTURE PARAMETERS FOR THE
ORTHOTROPIC BIMATERIAL INTERFACE CRACKS

CHYANBIN HWU
Institute of Aeronautics and Astronautics, National Cheng Kung University, Tainan, Taiwan 70001,
R.O.C.

Abstract—In order to define a proper bimaterial stress intensity factor, a general solution for the collinear
interface cracks between dissimilar anisotropic media is applied to the near-tip of an interface crack.
Through the operation of normalization, scaling, non-dimensionalization and transformation, a conve-
nable definition which can be reduced to the classical stress intensity factors for a crack tip in homogene-
ous media has been obtained. For special cases of orthotropic/bimaterial interface cracks, the explicit solutions
are derived in terms of engineering constants.

INTRODUCTION

Many of the problems concerning interface cracks have been solved in the literature; for example,
references may be found in a recent paper by the author [1]. Fracture parameters of interface cracks
are also widely discussed. However, due to the complexity of the problem itself, most of them do
not provide the explicit solutions in terms of engineering constants. For the purpose of engineering
applications, expressions by engineering constants are useful for the understanding of the physical
behavior. In this paper, by the use of explicit expressions for the Barnett–Løthe tensors obtained
by Dongue and Ting [2] and the proper definition of the bimaterial stress intensity factors presented
here, the near-tip solutions have been provided in terms of engineering constants.

FRACTURE PARAMETERS

A general solution for the interface cracks between dissimilar anisotropic media has been
obtained [1, 3] by applying Stroh’s formalism [4] and the method of analytical continuation [5]. As
stated by Hwu [1, 3], the full field solutions can be derived by finding

\[ \psi(z) = \frac{1}{2\pi i} X_0(z) \int_{z}^{\infty} \frac{1}{s-z} \left[ \frac{1}{X_0^2(s)} \mathbf{b}(s) \right] ds + X_0(z) p(z), \]

(1)

where a prime (') denotes differentiation with respect to its argument; the integration path \( \Gamma \) lies
along the region of cracks; \( \mathbf{f}(z) \) is the self-equilibrated prescribed traction applied on the upper
and lower surface of the crack; \( p(z) \) is an arbitrary polynomial vector with a degree no higher than
the number of cracks \( n \), which may be determined by the infinity conditions and the single-valuedness
requirement of displacements. \( X_0(z) \) is the basic F Cleaner function matrix defined as

\[ X_0(z) = A \Gamma(z), \]

(2a)

where

\[ A = [a_{ij}], \quad \Gamma(z) = \left[ \prod_{k=1}^{n} (z - a_k)^{-\alpha_k} \right] \mathbf{b}(z). \]

(2b)

The angular bracket \( \langle \rangle \) stands for the diagonal matrix, i.e. \( \langle f \rangle_s = \text{diag}[f_1, f_2, f_3] \), in which
each component is varied according to the Greek index \( s \), and \( \alpha_s, s = 1, 2, 3 \), of (2b) are the
eigenvalues and eigenvectors of

\[ (M^{*} + \epsilon^{*} \mathbf{M}^{*}) \mathbf{b} = 0. \]

(3a)

The explicit solution for the eigenvalue \( \delta \) has been given by Ting [6] as

\[ \delta_s = -\frac{1}{2} + \epsilon_s, \quad s = 1, 2, 3, \]

(3b)