PLANE PROBLEMS FOR ANISOTROPIC BODIES WITH AN ELLIPTIC HOLE SUBJECTED TO ARBITRARY LOADINGS

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Abstract: The two-dimensional elliptic hole problems have been widely studied and many important results have been obtained. Among several formulations for anisotropic elasticity, Stroh's formalism has been proved to be powerful and elegant in solving two-dimensional problems. Recent discovery of identities enables us to convert the complex-form solution into a real form. The real form solutions do not contain the eigenvalues and eigenvectors of material constants. Hence, it avoids the problem of repeated eigenvalues and can be applied to any degenerated materials. Furthermore, the present two-dimensional formulation includes not only inplane but also antiplane problems and the problems where inplane and antiplane deformations couple each other. Recently, the above features are presented for the case of uniform loading. In this paper, an arbitrary loading condition is studied. By expressing the arbitrary loading in terms of the Fourier series, an explicit closed form solution has been obtained in the form of an infinite series. To verify our results, some special loading conditions are considered. To show the generality, an example of pin-loaded hole is given. Finally, the crack problems are studied and the stress intensity factors of the crack subjected to arbitrary loading are given.

Keywords: Anisotropic, Elliptic Hole, Arbitrary Loading, Stroh's formalism, Crack.

INTRODUCTION

The problem of determining stress distribution induced by holes has aroused considerable interest for almost half a century. Of various holes, the elliptic shape has evolved the most interest among researchers. A brief introduction of the history of research on this subject was given in [1]. Following the most recent work [1], which considered two-dimensional problems of the anisotropic elastic solid with an elliptic inclusion subjected to uniform loading at infinity, we extend the problem to arbitrary loading conditions by applying the Stroh formalism [2]. Similar work has been done independently by Ting [3]. By careful derivation, one may prove that the solutions shown in [3] are equivalent to the present one for the general arbitrary loading conditions. However, no examples about the general loading conditions and cracks, which are important for practical applications, are presented in [3].

Similar to the case considered in [1], the present solution are valid for any kind of anisotropic materials including the degenerated materials such as isotropic materials. Moreover, they are valid for both inplane and antiplane problems. The solutions can also be applied to crack problems by letting the minor axis of the ellipse tend to zero.

TWO-DIMENSIONAL ANISOTROPIC ELASTICITY

For two-dimensional anisotropic elasticity, there are two different formulations in the literature [2,4]. One is the Lehnitzuk's approach [4] which starts with the equilibrated stress functions then compatibility equations, the other is Stroh's formalism [2] which starts with the compatible displacements then equilibrium equations. The equivalence of these two formulations has been discussed by Sun [5]. In this paper, we follow Stroh's formalism and the notation employed in [1,6].

In a fixed rectangular coordinate system \(x_1, x_2 = 1, 2, 3\), let \(u_1, u_2, u_3\) be, respectively, the displacement, stress and strain. The strain-displacement equations, the stress-strain laws, and the equations of equilibrium are

\[
\sigma_3 = \frac{1}{2}(u_{2x_1} + u_{1x_2}),
\]

\[
\sigma_3 = C_{ijkl} u_{kx_l},
\]

\[
C_{ijkl} u_{kx_l} = 0
\]