Various Rigid Inclusions in Anisotropic Media

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ABSTRACT

The problems of two dimensional anisotropic plates containing various rigid inclusions have been widely studied. However, the exact solutions exist only when the shape of inclusion is ellipse or its geometric limits. This solution has been found for the shape of square, triangle, etc. etc. By using Stroh formalism, a unified expression for various rigid inclusions is obtained in this paper. The mapping function used in the derivation is nonconformal when the shape of inclusion is non-ellipse which will cause the discontinuity of displacements and stresses. However, if this discontinuity approaches to zero, the solution obtained will approximate an exact solution. Although the general solution is in complex form, the interfacial stresses along the boundary of inclusion can be obtained in real form through the use of identities developed in the literature. With this real form solution, the problem of repeated eigenvalues is avoided and the solution can then be applied to any kind of anisotropic materials.

INTRODUCTION

The problems of rigid inclusions occur frequently in many engineering applications. Stress concentration happens when a plate contains rigid inclusion, which will stimulate failure of the materials. The increased utilization of laminated composites has renewed the interests of researchers for the analyses of anisotropic materials. Of various shapes, the ellipse and its geometric limits such as circles and line inclusions have evoked the most interest among researchers, such as Ref. [3,4,6,7,10,12,14,17]. For the shapes differing from ellipse such as rectangles, triangles, etc., the exact solutions are available only for isotropic materials due to mathematical difficulty [2,5,11]. All these solutions have different expressions when the shape of inclusion or the properties of matrix are changed. By using Stroh formalism [13], Hwu [8] obtained a unified expression for any kind of anisotropic materials with various openings, which is exact for isotropic materials with any kind of openings and for anisotropic materials with elliptical openings. By following the procedure that Hwu [8] used for the opening problems, a unified expression for the anisotropic media containing various rigid inclusions has been obtained in this paper. The loading conditions considered are pure bending and uniform loading which includes biaxial loading, inplane or antiplane loading. The mapping function used in the derivation is nonconformal when the shape of inclusion is non-ellipse, which will cause the discontinuity of displacements and stresses. Because the function chosen is negative power of the argument, the discontinuity will approach zero at infinity. If the discontinuity along the entire x-axis approaches zero, the solutions obtained will then approximate exact solutions. Finally, the present results are verified by comparing the solutions given in the literature for some special cases [11,17].

RIGID INCLUSIONS

For two dimensional anisotropic elasticity, the general solutions for the stress functions $\phi$ and displacements $u$ can be written as [8]

$$
\phi = \phi^0 + 2Re \left( \sum_{k=1}^{n} \Omega_k (Z) \right) q \sigma \eta \eta^* 
$$

$$
\zeta = \zeta^0 + 2Re \left( \sum_{k=1}^{n} \Phi_k (Z) \right) q \eta \eta^* 
$$

$$
\sigma_k = \phi_{2k} \quad \eta_k = \phi_{1k} 
$$

where

$$
\Omega(Z) = \text{Re} \left[ \eta \left( t_1(x), x_2(x), y_2(x) \right) \right] 
$$

$$
\Phi(Z) = \text{Re} \left[ \phi \left( t_2(x), x_2(x), y_2(x) \right) \right] 
$$

Re stands for the real part, $\sigma$ and $\phi$ represents, respectively, the displacements, stresses and stress functions. $\sigma_1, \sigma_2, \sigma_3$ are the eigenvalues and eigenvectors of the matrix constants. $\Omega(\zeta)$ is an arbitrary function with argument $\zeta$, $\psi$ is a complex constant to be determined by the boundary conditions.

Consider an inclusion, the contour of which is represented by

$$
x_2 = a \left( \cos \psi + \varepsilon \cos k \psi \right) 
$$

$$
x_1 = a \left( \cos \psi - \varepsilon \sin k \psi \right) 
$$

where $0 < c \leq 1$, and $k$ is an integer. When $\varepsilon = 0$, $k$