Collinear cracks in anisotropic bodies

CHYANBIN HWU
Institute of Aeronautics and Astronautics, National Cheng Kung University, Tainan, Taiwan, 70101, Republic of China

Received 20 April 1990; accepted in revised form 9 February 1991

Abstract. A general solution for the stresses and displacements of collinear cracks in an infinite homogeneous anisotropic medium subjected to uniform loading at infinity has been given in this paper by using the Stroh's formulation. The solutions are valid not only for plane problems but also for antiplane problems and the problems whose inplane and antiplane deformations couple each other. Two special collinear crack problems are solved explicitly: (1) two collinear cracks, (2) an infinite row of evenly spaced collinear cracks. A closed form solution of the stresses and displacements in the entire domain is obtained. Through the use of identities developed in the literature, the stress intensity factors, crack opening displacements and energy release rate are expressed in real form, which are valid for any kind of anisotropic materials including the degenerate materials such as isotropic materials. The simple explicit form solutions for the crack opening displacements and energy release rate reveal that the effect of anisotropy is totally determined by the fundamental elasticity matrix $L$. The relation between the stress intensity factors and energy release rate is obtained in quadratic form and related to $L$.

1. Introduction

One of the standard crack problems is an infinite plate with an arbitrary number of collinear cracks. Although some results [1, 2] obtained by a Lekhnitskii approach [3, 4] reveal the general formulation for anisotropic problems. To the author's knowledge, there are no simple explicit form solutions for the crack opening displacements and energy release rate due to the mathematical complexity. Moreover, the general formulation of anisotropic problems cannot be specialized to those of isotropic problems due to the repeated eigenvalue of the elastic constants of isotropic materials, which also restricts the application to other degenerate materials [5]. The inplane and antiplane deformations cannot always be decoupled for generally anisotropic media, except for the local field near a crack tip [6]. However, the solutions presented in the literature [1, 2, 7] can only deal with inplane or antiplane problems individually.

Recent work [8–12] of Stroh's formulation [13] in anisotropic elasticity points out that the stress functions of isotropic problems may be directly applied to the anisotropic cases by a slight modification. For crack problems, it is also known [7] that the stress intensity factors of anisotropic problems are identical to those of corresponding isotropic problems if the loads on the crack surface are self-equilibrated. Therefore, a proper stress function for collinear cracks in anisotropic bodies may be chosen by referring to the corresponding isotropic problems [14–19]. The solutions are then obtained by satisfaction of the boundary conditions and the requirement of single-valuedness of displacements. Like previous work by Stroh's formulation [8–12], the solutions are valid not only for plane problems but also for antiplane problems and the problems whose inplane and antiplane deformations couple each other.

In this paper, two special collinear crack problems are solved explicitly. One is an infinite homogeneous anisotropic medium with two collinear cracks subjected to uniform loading at