GREEN'S FUNCTIONS OF TWO-DIMENSIONAL ANISOTROPIC PLATES CONTAINING AN ELLIPTIC HOLE

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Abstract—For a two-dimensional anisotropic plate, the Green's function satisfying traction-free boundary conditions around an elliptic hole is developed using Stroh's formalism. A combination of this function and the boundary element method shows that it is the most effective approach for solving hole problems. The generality of the present Green's function is shown by the broader meaning of the following words: "Two-dimensional" includes not only in-plane but also anti-plane problems and the problems where in-plane and anti-plane deformations couple each other. "Anisotropic", which need not have any material symmetry restrictions, means that it covers the solutions given in the literature, which only deal with orthotropic or monoclinic materials. "Elliptic" includes the special case where the minor axis of the ellipse tends to zero, i.e., the case of a Griffith crack. The accuracy of the numerical method presented is then verified by comparisons with exact or accepted solutions of several examples, such as an infinite or a finite plate with an elliptic hole or a crack under in-plane or anti-plane loading. The materials used are isotropic, orthotropic or lami-nated composites. Finally, problems where the hole boundary is not traction-free are solved, such as with inclusions and pin-ended holes.

INTRODUCTION

Most practical structures contain holes as parts of basic design. However, such holes cause high stress gradients, which have been studied by many investigators in the past. Analytic solutions for infinite anisotropic plates containing elliptic holes under uniform loading can be found in Svinin (1961) and Lekhnitskii (1968). Due to the difficulties in satisfying the boundary conditions for finite plates, numerical methods such as the finite element and boundary element method are now widely used. The basis of the boundary element method is Green's function (or fundamental solution). The Green's function for infinite anisotropic plates was presented by Green (1941), where the boundary condition along the elliptic hole was not satisfied. In the work of Tarn and Chen (1987) and Kamel and Liaw (1989a,b), the Green's function obtained by using Lekhnitskii's approach was improved in order to satisfy the elliptic hole boundary condition. Nevertheless, the solutions obtained are valid only for monoclinic materials and in-plane loading.

In this paper, the Green's function satisfying the traction-free condition on an elliptic hole in an infinite anisotropic plate is derived in closed and compact forms using Stroh's formalism (Stroh, 1958; Hwu and Ting, 1989). The derivation is valid for general anisotropic materials which need not have any material symmetry restrictions. The out-of-plane components of displacements and stresses are generally nonzero in Stroh's formalism. Hence the present solutions are valid not only for plane problems but also for anti-plane problems and problems whose in-plane and anti-plane deformations couple each other. By letting the minor axis of the elliptic hole tend to zero, the Green's function for the corresponding crack problems and its associated stress intensity factors are obtained explicitly.

A boundary element method employing the Green's function derived in this paper is used to analyze the stress distribution of finite plates containing traction-free elliptic holes. The boundary conditions along the elliptic hole are satisfied by the Green's function so that it is unnecessary to include the elliptic surface in the boundary integration. This explains why the present method is more effective than other numerical schemes. This concept has been widely used in the analysis of isotropic or anisotropic plates with cracks (Snyder and Crowe, 1975; Crowe, 1978; Murakami, 1978; Clements and Haskelgrove, 1983; Ang and

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