Anisotropic Plates With Various Openings Under Uniform Loading or Pure Bending

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1 Introduction
The problem of determining areas distributions induced by openings has aroused considerable interest for almost half a century. Of various openings, the elliptic shape and its geometric limits, such as circles and slits, have evoked the most interest among researchers. If the stress concentration is of more concern, optimization in a class of hole shapes (Durelli and Rajajia, 1979; Díaz, 1981) has shown that the optimized hole is not necessary an ellipse. In the literature, however, there are very few works dealing with openings differing from an ellipse. Evans-Iwanski (1956) used the complex variable approach developed by Pickard and Parshley (1935) to derive the stress solutions for an infinite isotropic plate with a triangular hole. A similar approach has been applied to the problem of an isotropic plate with rigid rectangular indentations by Yau (1952) and Chang and Conway (1968). For orthotropic plates with rectangular openings, work has been done by Yong (1981), Rajajia and Nalls (1983). Their studies were based on the solutions given by Liberski (1980), which are only approximate solutions due to the mathematical difficulties.

In Lekhnitskii's approach (1968), the state of generalized plane stress or plane strain is the restrictive condition. This implies that at each point of the anisotropic plate there is a plane of elasticity symmetry which is parallel to the middle plane of a thick plate, or normal to the generator of a long cylinder. Therefore, the solutions obtained by using this approach are valid only for monotropic materials, and in-plane loading will not induce antiplane displacement or transverse shear. Furthermore, a special treatment is also required for degenerate anisotropic elastic materials. However, for the general anisotropic materials it is not always possible to treat the in-plane and anti-plane deformations independently, and this approach cannot be applied.

In my recent work (Ting and Ting, 1989), an analytical solution for the anisotropic plate with an elliptic inclusion subjected to uniform loading at infinity has been obtained using the Stroh formalism (Bekitezhi et al., 1993; Stroh, 1959; Barnett and Lothe, 1973 and 1975; Chadwick and Smith, 1975; Ting, 1986) instead of the classical formulation (Lekhnitski, 1963 and 1968). The results are valid for any kind of anisotropic materials including monotropic materials and degenerate materials such as anisotropic materials. In the case when the anisotropic plate contains a triangular, oval, or square opening, the exact solution has not yet been found. The only analytical solution available in the literature is an approximate solution for orthotropic plates (Lekhnitski, 1980). To find solutions valid for general anisotropic plates, the Stroh formalism has been applied. In order to obtain an exact solution for various openings, we use the technique of conformal mapping. Instead of considering the opening to be slightly different from an ellipse and introducing a small parameter which will characterize the deviation of the opening from an ellipse, we treat the opening as itself. In the former case, the stress function is approximated by using that of the elliptic opening, due to the difficulties of finding the exact and explicit form. In the

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