Buckling and Postbuckling of Delaminated Composite Sandwich Beams

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Consider a sandwich plate with balanced or unbalanced anisotropic composite laminated faces and an ideally orthotropic honeycomb core. The paper presents elastic buckling and postbuckling analysis of an axially loaded plate with an across-the-width delamination symmetrically located at the interface of the upper face and core. Since the plate undergoes cylindrical bending deformations on the postbuckling states for the cases considered, only one-dimensional formulations are employed. The inverse-deflection and bending moment of the postbuckling solutions are obtained by applying the one-dimensional formulations. The explicit closed-form expressions of the critical buckling load and energy release rate are derived based on this postbuckling solution. Because there is no such general solution presented in the literature, verification is done by some special cases such as delaminated composites (without core), perfect sandwiches (without delamination), and thin-film delaminations. The effects of core, face, and delamination length on the buckling load, the delaminative growth, and the ultimate axial load capacity of the delaminated composite sandwiches are also discussed in this paper.

Nomenclature

\[ A_i = \frac{1}{2}(A_{i1} + A_{i2}) \]
\[ B_i = B_1 \]
\[ C_i = \text{extensional stiffness of laminated composite} \]
\[ D_i = \frac{B_{i1}}{A_{i2}}, \quad i = 1,2 \]
\[ H_i = \text{coupling stiffness of laminated composite} \]
\[ G_i = \text{core thickness of sandwich} \]
\[ D_i = D_1, D_{i2} = D_2, \quad i = 1,2,3 \]
\[ D_i = \text{coupling stiffness of laminated composite} \]
\[ E_i = \text{Young's modulus in } z \text{ direction} \]
\[ f = \text{face thickness when two faces are of equal thickness} \]
\[ f_{i1} = f_{i2} = \text{thicknesses of upper and lower faces} \]
\[ G = \text{energy release rate} \]
\[ G_{	ext{trans}}, G_{	ext{int}} = \text{transverse shear moduli in } x-z \text{ and } p-z \text{ planes} \]
\[ G_0 = \text{critical energy release rate} \]
\[ I = \text{half-length of composite sandwich} \]
\[ M_i = \text{bending moment of region } i, i = 1,2,3 \]
\[ N_{i1}, N_{i2}, N_{i3} = \text{resultant moments} \]
\[ N_i = \text{resultant forces} \]
\[ n = \text{number of layers including upper and lower faces, Eq. (6)} \]
\[ P = \text{compressive axial force applied at the ends} \]
\[ P_r = \text{buckling load of composite sandwich} \]
\[ P_{c1} = \text{buckling load of composite laminate} \]
\[ P_{c2} = \text{compressive axial force of region } i = 1,2,3 \]
\[ P_i = \text{at critical buckling load} \]
\[ Q_{i1}, Q_{i2} = \text{transformed reduced in-plane stiffness of a single layer} \]
\[ Q_{i3} = \text{resultant transverse shear forces} \]
\[ l = \text{extreme transverse shear stiffness} \]
\[ L = \text{strain energy} \]
\[ W_i, W_j = \text{displacements in } x, y, \text{ and } z \text{ directions} \]
\[ W_{i1}, W_{i2}, W_{i3} = \text{displacements of region } i \]
\[ W_{i1}, W_{i2} = \text{displacements of the plane } z = 0 \]

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1. Introduction

**SANDWICH** construction has been used in aeronautical applications for more than 40 years, since it has many advantages, for example, high bending stiffness, good weight savings, good surface finish, good fatigue properties, good thermal and acoustical insulation, etc. Today, there is a renewed interest in using sandwich structures due to the introduction of new materials such as laminated composites for the faces of sandwich panels, which offer a long awaited capability with both high stiffness and low specific weight. Similarly, new materials for the core are now available, such as nonmetallic honeycombs and plastic foams. However, these new materials also induce some new problems that need to be solved. One of the most frequently encountered problems in composite laminates is interfacial cracking, sometimes known as delamination. For composite sandwiches, there is an extra interface between the face and core that may be weaker than those in layered composite faces. Delaminations may occur due to a variety of reasons such as low energy impact, manufacturing defects, or high stress concentrations at geometric or material discontinuities (e.g., the well-known free-edge effects). The presence of delaminations is of major concern, especially in compressively loaded components where delaminations may grow under fatigue loading by out-of-plane distortion.

Received July 8, 1991; revision received Nov. 22, 1991; accepted for publication Nov. 22, 1991. Copyright © 1992 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved. *Associate Professor, Institute of Aeronautics and Astronautics. **Graduate Research Assistant, Institute of Aeronautics and Astronautics.
\[ P_r = \frac{(D_1 + D_2 - D_3)}{2} \times \left[ \frac{g}{\lambda_3 \tan \alpha \pi} + \frac{k}{\lambda_3 \tan \alpha \pi} \right] \]  

(18a)

where

\[ k = \frac{A_1}{A_2 + A_3} \]
\[ \lambda_3 = \frac{P_{sr}}{D_1 - (1 - \frac{P_{sr}}{S})} \]
\[ \frac{P_{sr}}{D_3} = \frac{P_{pl}}{D_2} = \frac{(1 - \frac{P_{sr}}{S})}{D_1} \]

Note that the equality \( (B_1 - B_2) \lambda_3 (\alpha - 1) = D_3 + D_1 \)

which can be derived directly from Eq. (14c), has been used.

IV. Energy Release Rate

If the growth of a buckled delamination is governed by a Griffith-type criterion of a critical energy release rate, the prediction of whether delamination will grow requires an evaluation of energy release rate \( G \). By applying the previous postbuckling solution, evaluating the total potential energy and differentiating the result with respect to the delamination length \( 2a \), we may obtain an explicit expression for \( G \). The function \( G(a) \) can then be used to study the initiation and stability of delamination growth.

The total potential energy of the buckled delaminated sandwich consists of the contributions from strain energy \( U \) due to bending and stretching at the faces, shear energy of the core, and the potential energy \( W \) of the external forces, i.e.,

\[ \Pi = U - W \]

(19a)

where

\[ U = \frac{1}{2} \int \left( M_{x}^{2} + N_{x}^{2} + Q_{s}^{2} \right) \text{d}x \]
\[ W = -\int P_{pl} \text{d}x \]

(19b)

Note that the factor 2 is due to the symmetry condition considered in our problem. The expressions for \( M_{x}, N_{x}, N_{s}, N_{pl} \), and \( Q_{s} \) can be found as the second equation of Eq. (14b); the first equations of Eqs. (4b), (4a), and (7); and Eq. (13). An alternate expression for \( W \) is

\[ W = -2 \int \left( P_{pl} u^{||} \right) - u^{||}(p) + P_{pl} u^{||}(p) \]

(18c)

where the superscript (i) denotes the region number. By using the postbuckling solutions given in Eqs. (13-15), the final simplified result of the total potential energy can be expressed as

\[ \Pi = A_1 P_1^2 / 2 a + A_2 P_2^2 / 2 \beta \left( \delta + P_{pl}^2 \right) \sin \lambda_3 \sin (\alpha - \delta) \]

(20a)

where

\[ \alpha = \frac{1}{2} \left( A_1 P_1^2 + A_2 P_2^2 + A_3 P_3^2 \right) \]
\[ \beta = \frac{1}{2} \left( B_1 P_1 + B_2 P_2 + B_3 P_3 \right) \]

(20b)

The energy release rate \( G \) is then calculated as

\[ G = \frac{\partial \Pi}{\partial a} \]

(21a)

\[ = -2 \int \left( P_{pl} u^{||} \right) - u^{||}(p) + P_{pl} u^{||}(p) \sin \lambda_3 \sin (\alpha - \delta) \] \times \left[ \frac{\lambda_3 \sin \lambda_3 (\alpha - \delta)}{\sin (\alpha - \delta)} \right] \]

where the prime (·) denotes differentiation with respect to the half delamination length \( a \). From Eq. (13), we can also show that

\[ \lambda_3 \sin \lambda_3 (\alpha - \delta) \]

(21b)

V. Special Cases

A. Delaminated Composite (Without Core)

The problem of delamination buckling in composite laminates has received a considerable amount of attention. Analytical investigation by a one-dimensional model has been done by several researchers. The present results of composite sandwich are also applicable to the cases of composite laminates (without core) by letting the terms consisting of \( 3 \sin \lambda_3 \). The expressions for the postbuckling deflections and buckling load are exactly the same as Eqs. (13), (15), and (16) except that \( \lambda_3 \) and \( \lambda_1 \) are now replaced by \( \lambda_3 = P_{pl}/D_3 \) and \( \lambda_1 = P_{pl}/D_1 \). The energy release rate shown in Eq. (21) is then simplified to

\[ G = \frac{1}{2} \left( A_1 P_1^2 + A_2 P_2^2 + A_3 P_3^2 \right) \frac{1}{2 \beta} \left( \frac{\lambda_3 \sin \lambda_3 (\alpha - \delta)}{\sin (\alpha - \delta)} \right) \]

(22)

An algebraic expression for the energy release rate obtained by means of the path-independent \( J \) integral has been presented by Sallam and Simitses. That expression looks complicated and different from the one given in Eq. (22). However, by careful deduction one can prove that they are exactly the same.

B. Perfect Sandwich (Without Delamination)

By letting the delamination length \( 2a \) approach zero, the characteristic equation for buckling load shown in Eq. (18) can be reduced to

\[ \frac{\pi}{2} \cot \lambda_3 \tan \lambda_3 (\alpha - \delta) = \frac{D_1}{P_{pl}} \]

(23)

The solution to Eq. (23) exists only when \( \lambda_3 = \pi/\alpha \). From the second equation of Eq. (18b) we have

\[ P_{pl} = \frac{D_1 \pi^2}{4 (D_2 S)(\pi \alpha)} \]

(24)

which is a well-known solution for a perfec sandwich. The buckling load for a perfect sandwich may be considered as an upper bound for the axial load capacity of a delaminated sandwich.
C. Thin-Film Delamination

Because the faces are relatively thinner than the depth of the core, the delamination laid on the interface can usually be treated as a thin-film delamination. Because the delamination is relatively slender in comparison with the whole plate, the buckling may be initiated by local buckling of the thin delamination. Since the thin layer of delamination has elastically supported ends, the buckling load \((1 - k)P_{cr} = \frac{4E_2D_2}{\pi^2a^4}\) is close to but less than that of a fixed end plate of length \(2a\), i.e.,

\[
(1 - k)P_{cr} = \frac{5D_2}{1 - \frac{2}{\pi^2a^4}}
\]

or

\[
\sigma_t = \pi \nu\frac{P}{a^2}
\]  

(25)

Consider the case when \(\lambda = \pi \nu\); the postbuckling deflection shown in Eq. (13) can be reduced to

\[
\omega = \omega_0 = 0
\]

(26a)

where the amplitude \(\Gamma_s\) is related to \(\Gamma\) by

\[
\Gamma_s = \frac{\lambda_0 \sin \lambda_0 (l - a)}{\lambda_0 \sin \lambda_0 a}
\]  

(26b)

The equilibrium and compatibility conditions obtained in Eq. (15) can also be simplified to, by using \(\lambda_0 = \pi \nu\) and Eq. (26b),

\[
\frac{P}{4D_2} = -A_2P_2 - (A_1 + A_3)P_3 = 0
\]

(27a)

where

\[
P_s = \frac{3\lambda_0 \pi^2}{4a^2}
\]  

(27b)

Equations (27a) and (27b) yield the unknown amplitude \(\Gamma_s\) and the required postbuckling load \(P_s\) to attain the special postbuckling deflection shown in Eq. (26). The energy release rate given in Eq. (23) for this special postbuckling deflection can also be reduced to

\[
G = \frac{\pi^2}{2D_2} \left( A + \frac{P_s^2}{3} \right)
\]  

(28)

If we consider 2 homogeneous orthotropic plate of thickness \(t\) containing a parallel plane crack at a depth \(d\) from the top surface of the plate, the previous results in Eqs. (25–27) can be shown to be exactly the same as those presented by Yin et al.4

Similar to the cases discussed in Ref. 3, a lower bound of the ultimate load capacity is given by the combined axial load capacity of both detached plates.

VI. Results and Discussions

In the following examples, graphite/epoxy lamina is selected to construct the faces of sandwich whereas aluminum honeycomb is used to be the core of the sandwich. The half-length of sandwich beam \(l = 50\) mm. The material properties of graphite/epoxy are \(E_I = 181\) GPa, \(E_2 = 10.3\) GPa, \(G_{12} = 7.17\) GPa, and \(\nu_{21} = 0.28\) where \(E\), \(G\), and \(\nu\) are the Young's modulus, shear modulus, and Poisson's ratio, respectively.

The subscript 1 denotes the fiber direction and the subscript 2 denotes the transverse direction. The thickness of each lamina by \(0.125\) mm. The reference effective transverse shear modulus of aluminum honeycomb selected are \(G_{12} = 0.146\) GPa, \(G_{13} = 0.0994\) GPa, and the thickness of honeycomb core \(t = 10\) mm.

A. Core Effect on Buckling Load

The effect of core on the buckling load is usually discussed by considering the thickness and effective transverse shear modulus separately. However, in our formulation only one parameter \(S_1 = c_2G_{13}\) needs to be considered. An example of \([0/\theta/90/\theta]/_{5}h_{\text{honeycomb}}\) with \(S_1 = 1.46\) MN/m is used to study this effect. The results are presented by the relative buckling load \(P_{cr}/P_{cr}^0\) where \(P_{cr}^0\) denotes the buckling load of delaminated composite (without core). Figure 2 shows that the
Table 1 Effect of stacking sequence on buckling load of delaminated sandwich (composite laminate/honeycomb)

<table>
<thead>
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<th>θ</th>
<th>P₀/𝑃₃₀</th>
<th>θ₀</th>
<th>θ₀°</th>
<th>θ₀0°</th>
<th>θ₀°⁰</th>
<th>θ₀⁰°</th>
</tr>
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<tr>
<td>0.2</td>
<td>0.2598</td>
<td>0.1772</td>
<td>0.1484</td>
<td>0.1476</td>
<td>0.1311</td>
<td></td>
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<tr>
<td>0.3</td>
<td>0.1515</td>
<td>0.0788</td>
<td>0.0690</td>
<td>0.0656</td>
<td>0.0503</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.0650</td>
<td>0.0431</td>
<td>0.0371</td>
<td>0.0349</td>
<td>0.0283</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.0416</td>
<td>0.0254</td>
<td>0.0238</td>
<td>0.0236</td>
<td>0.0181</td>
<td></td>
</tr>
</tbody>
</table>

*θ = [0°, −45°/45°/0°], θ₀ = [−45°/−45°/0°], θ₀0° = [45°/−45°/0°], and θ₀°⁰° = [−45°/−45°/−45°].

![Fig. 4 Effect of delamination length on buckling load.](image)

![Fig. 6 Energy release rate vs applied compressive force with delaminated length given.](image)

The buckling load will increase when the transverse shear stiffness S increases until it reaches a certain value and becomes a constant, which can be explained as thin-film delamination buckling, and is verified by Eq. (25). The solid line is under the condition $G_{23} = G_{30}$ with $c$ varied whereas the dot line is under the condition $c = 45°$ with $P_{30}$ varied. It shows that no matter how we change the transverse shear stiffness by varying core thickness c or transverse shear modulus $G_{23}$, the buckling loads for both cases are almost the same. The difference comes from $D_{11}$, $D_{22}$, and $D_{44}$ in Eq. (18), which are related to the core thickness but are independent of the transverse shear modulus. Figure 2 also gives us a minimum value of $S$ than makes the buckling load of delaminated sandwich larger than that of delaminated composite. Below this value, it may not be a useful construction for sandwich since the core thickness may be small (thin face thickness and the usual assumption for sandwich listed in Sec. V). It may not be valid. One should note that the special case discussed in Sec. V is avoided by letting the term containing $S$ vanish, not by letting $S$ be zero.

B. Face Effect on Buckling Load

To discuss the effect of face, we consider the thickness, fiber direction, and stacking sequence of the laminated composite. Before the calculation, we may expect that buckling load will increase when the face thickness increases, or when the fiber is oriented to the direction of load (i.e., $\theta$ varies from 90 to 0 deg). Figure 3 verifies this expectation. A series of composite sandwiches (90°/90°/honeycomb), are used in this case. The reference parameters are $P_{30} = 0.625$ kN (i.e., $\theta = 45°$, $\theta_{0} = 90$ deg, and $e = 0.22$); and $P_{30}$ is the buckling load corresponding to $f = f_{0}$ with $\theta = 0$ deg. The thickness effect is considered by varying the number of $\theta = 0$ deg laminates, i.e., changing $e$ from 0 to 4.

To study the effect of stacking sequence, we consider five different sequences shown in Table 1 where $P_{30}$ represents the buckling load of perfect composite sandwich. For different delamination length, Table 1 shows that the buckling load of delaminated sandwich with stacking sequence $\theta_{0}$ is always the highest one, whereas that of $\theta$ is always the lowest one. However, $\theta_{0}$ is not the one with highest bending stiffness $D_{22}$ and $D_{44}$ is not the one with lowest bending stiffness, the effect of coupling stiffness $B_{0}$ is revealed in this example. The order of the buckling load shown in this table is $\theta = 0 > \theta = 2 > \theta = 3 > \theta = 4 > \theta = 5$, which is consistent with the order of $D_{22}$ (or $D_{44}$), not $D_{0}$.

This also means that the example considered can be approximated by the thin-film delamination as shown in Eq. (25).

C. Effect of Delamination Length on Buckling Load

The sandwich construction ($\theta_{0}/90°/\text{honeycomb}$) is used as an example to study the delamination length effect on buckling load. Figure 4 shows the relation between the relative buckling load $P_{0}/P_{30}$ and delamination length $a$ where $P_{30}$ denotes the buckling load of perfect sandwich. As expected, the upper bound solution is close to the buckling load of perfect sandwich shown in Eq. (24), whereas the combined total load of two completely detached beams represents the lower bound. Figure 4 also shows that the buckling loads for some short delaminations are a little higher than $P_{30}$, which seems unreasonable at a first glance but the same phenomenon has been found in experiments for delaminated composites.$^{15,16}$

D. Delamination Growth

The postbuckling analysis assumes that the fracture toughness of the material is sufficiently large to resist the growth of delamination. When delamination growth does occur, the geometry of the problem is inevitably changed. However, owing to the intrinsic complexities involved, such as anisotropy, inhomogeneity, and noncontinuity, a satisfactory, physically meaningful, and universal delamination failure criterion has not yet been found. Here, we assume that delamination growth is governed by a critical total energy release rate $G_{0}$. For a
Fig. 6 Energy release rate vs. delamination length with applied compressive force given.

specified delamination length, the buckling load \( P_b \) can be obtained from Eq. (18), and the associated energy-release rate immediately after buckling is calculated in Eq. (21). If the energy release rate is less than the critical value \( G_c \), the delamination will not grow and the ultimate axial load capacity \( P_{ul} \) of the delaminated sandwich will be larger than \( P_b \), otherwise \( P_{ul} = P_b \). However, the actual \( G_c \) is still unknown. For the purpose of simplifying the delamination growth, we take \( G_c \) as the critical total energy-release rate of the composite laminae used in the sandwich face. In our examples, the sandwich is of the type \([0/45/-45]_2s\) honeycomb, and the faces of sandwich considered are \([0/45\] or \[45/-45\]), made of T300/5208 graphite epoxy whose critical value \( G_c = 400 \, \text{kJ/m}^2 \) Ref. 19 and \( E_T = 137.7 \, \text{GPa} \), \( E_M = 14.5 \, \text{GPa} \), \( G_{TM} \approx 5.9 \, \text{GPa} \), \( v_{TM} = 0.3 \), and \( t_F = 0.14 \, \text{mm} \). Figure 5 shows that, for a given delaminating, the energy-release rate increases when the applied axial load increases in which \( P_b \) is the buckling load of perfect sandwich. Generally, each curve for a fixed \( \alpha \) has two steep slopes. One is at the beginning, which corresponds to the buckling load; the other is near the end, which may be responsible for the delamination growth. For example, if \( \alpha = 0.7 \), there is a threshold value of \( P/P_b = 0.173 \) below which the energy-release rate is identical to zero. This means that the threshold value of \( P \) is the buckling load \( P_b \). The energy-release rate associated with the load immediately after buckling is approximated to \( 0.2G_c \) which is less than the critical value \( G_c \). Hence the delamination will not grow until the load is increased to \( 0.23P_b \), where the energy-release rate is \( G_c \) and \( P_{ul} = 0.23P_b \), \( G_c \) is the deflections of both upper and lower segments at the still flat. However, a small disturbance will make the sandwich buckle. When constant load increment \( 0.05P_b \) applies, the deflection of region 3 is convex upward whereas that of region 2 deforms a little. There are two stages that have great jump in the deformation. One occurs near the critical buckling stage and the other happens when \( P = 0.42P_b \) from which the energy release rate increases rapidly. This phenomenon reveals that the actual \( G_c \) may be higher than the chosen one.

Fig. 7 Postbuckling deflection of the delaminated composite sandwich with \( \alpha = 0.7 \).

VII. Conclusions

A mathematical model for the delaminated composite sandwich has been developed in this paper. By applying the one-dimensional formulations, the transverse deflection and bending moments of the postbuckling solutions are obtained. The explicit closed-form solutions of the critical buckling load and energy release rate are also derived based on this postbuckling result. Some special cases such as delaminated composite, perfect sandwiches, and thin-film delaminations are used to verify our solutions. The effects of core, face, and delamination length to the buckling load, the delamination growth, and the ultimate axial load capacity of the delaminated composite sandwiches are also discussed in this paper. The results show the following. 1) The buckling load will increase when the transverse shear stiffness increases until it reaches a certain value and becomes a constant. 2) The buckling load will increase when the face thickness increases or the fiber is oriented in the direction of load. 3) The upper bound solution of the delaminated composite sandwich is approximated by the buckling load of perfect sandwich whereas the combined axial load of two completely detached beams represents the lower bound. 4) If the energy release rate associated with the load immediately after buckling is less than the critical energy release rate, the delamination will not grow and the ultimate axial load capacity \( P_{ul} \) of the delaminated composite sandwich will be larger than the buckling load \( P_b \) of the perfect sandwich.

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References
