ANALYSIS AND IDENTIFICATION OF CRACKS
IN PIEZOELECTRIC MATERIALS

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ABSTRACT

To analyze the electromechanical behavior of the piezoelectric materials, a complex variable formulation of the Stroh formalism dealing with the two-dimensional anisotropic linear piezoelectric media has been applied. The analysis of cracks in piezoelectric materials is then performed by a special boundary element which embeds an analytical solution (of infinite domain problems) found by the Stroh formalism. For the purpose of identifying a crack in piezoelectric materials, the techniques of nonlinear optimization and artificial neural network are employed in this paper. The parameters used in the present study are the opening, shearing, tearing and electric mode stress intensity factors. The connection between these parameters and the exterior measured boundary data is provided by the boundary element algorithm.

INTRODUCTION

Due to the rapid development of intelligent space structure and mechanical system, advanced structures with integrated self-monitoring and control capabilities are increasingly becoming important. It is also well known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. Because of this intrinsic coupling phenomenon, piezoelectric materials are widely used as the sensors and actuators in intelligent advanced structure design.

No matter how carefully the load-carrying structural system (such as aircraft structures, turbines and rotors, etc.) is designed, there are instances of cracks or structural damage occurred during service. Since the cracks may grow under fatigue loading and the catastrophic damage may occur after a critical crack length, the periodical inspection during their operating life becomes necessary. To avoid any further destruction, one may wish to know in a non-destructive manner the location and the size of the crack. Indeed, such information about the crack geometry is indispensable for estimating the safety of the structure. In engineering practice, one usually carries out measurements, using techniques such as ultrasonics, magnetic flux leakage, X-rays, penetrant, eddy current, etc.[1], on the exterior boundary of the body to locate the crack. Recently, Qian, et.al [2], Rizos, et.al [3], and Shen and Taylor [4] identified the crack location and magnitude of a cantilever beam from the vibration modes. Nevertheless, much work is
still required in improving accuracy and efficiency for further practical application. The present paper will start with the precise crack identification in elastostatics for piezoelectric materials.

The determination of the unknown internal crack in a structure for which the boundary conditions are prescribed (measured experimentally), belongs to the category of inverse problems in mechanics. Since the measured data are usually provided on the exterior boundary, the boundary integral technique [5] for the analysis of the forward mechanics problems becomes a promising computational tool, e.g., ([6], [7], [8]). In contrast with the other materials, there have been relatively few research efforts on electro-mechanical analysis of cracks in piezoelectric materials. Analytical work by complex variable methods in elasticity may trace back to Barnett and Lothe [9] who treated dislocations and line charges in anisotropic piezoelectric insulators. Thereafter, some works related to the crack problems were done by Sosa and Pak [10], Suo, et.al [11], etc. Recently, by combining the extended version of Stroh formalism for the analysis of piezoelectric materials ([12],[9]) and the boundary element method, we developed a special boundary element for cracks in anisotropic piezoelectric materials [13], which may now be employed for the analysis of the forward mechanics problems.

As far as the inverse problems are concerned, the techniques of nonlinear optimization [14] and neural networks [15] are applied in this paper. Examples of the application of these techniques to crack detection may be found in ([4], [16], [17], [18]). In this paper, the parameters used for the identification are the well-known stress intensity factors which are calculated by the special boundary element developed in [13]. However, since the stress intensity factors are insensitive to the crack location, the attention of present study will be focused on the identification of crack size not its location.

CRACK ANALYSIS

The purpose of this study is to explore the feasibility of identifying a crack by using only the data from static deformation. To be successful in identification, we should be very careful about the choice of detectors which need to be sensitive to the crack size and location. Without making too much effort in searching for these parameters, we firstly try the well known parameters used in fracture mechanics, i.e., the stress intensity factors \( K \) which are defined as

\[
\begin{bmatrix}
K_{II} \\
K_{I} \\
K_{III} \\
K_{IV}
\end{bmatrix} = \lim_{r \to 0} \sqrt{2\pi r} \begin{bmatrix}
\sigma^{12} \\
\sigma^{22} \\
\sigma^{33} \\
D_2
\end{bmatrix},
\]

(1)

In eqn.(1), \( K_I, K_{II}, K_{III} \) and \( K_{IV} \) are, respectively, the opening, shearing, tearing and electric mode stress intensity factors; \( \sigma_{ij} \) and \( D_i \) are the stresses and electric displacements near the crack tip; \( (r, \theta) \) is the local coordinate with the origin at the crack tip and the direction \( \theta = 0 \) opposite to the crack surface.

In order to calculate the stress intensity factors, we need to know the stress distribution for a loaded plate containing a given crack. Consider a two-dimensional anisotropic linear piezoelectric medium \( \Omega \) bounded by the external boundary \( \Gamma_e \) and the internal crack boundary \( \Gamma_c^* \). The deformation field is described by the following governing equations and boundary conditions [13];

\[
C_{ijkl} u_{k,sj}(z) = 0, \quad z \in \Omega,
\]

(2a)
\[ u_k(z) = u_k^0, \quad z \in \Gamma_{eu}, \]
\[ t_k(z) = t_k^0, \quad z \in \Gamma_{et}, \]
\[ t_k(z) = 0, \quad z \in \Gamma^*_e, \]

(2b)

where \( i, j, k, s \) are ranged from 1 to 4; repeated indices imply summation, a comma stands for differentiation; \( C_{ijkl}^r \) are the augmented elastic constants including the elasticity tensor \( C_{ijkl} \), the piezoelectric stress tensor \( e_{kl} \) and the permittivity tensor \( \epsilon_{ij} \); \( u_k \) and \( t_k \) denote, respectively, augmented displacements and tractions in which the first three components are displacements and tractions, and the fourth one is related to the electric field; \( u_k^0 \) and \( t_k^0 \) are the prescribed values of \( u_k \) and \( t_k \); \( \Gamma_{eu} \) and \( \Gamma_{et} \) are the portions of the exterior boundary with specified \( u_k \) and \( t_k \), respectively; \( \Gamma_e = \Gamma_{eu} + \Gamma_{et} \); and \( z \) is the position vector.

Usually the above boundary value problems are solved by finite element method [19] in which the mesh of the entire body is necessary. For a problem that only the response of the outer boundary is concerned, the boundary element method [5] may become more attractive. Since the measured data are usually provided on the exterior boundary for the crack identification problems, the boundary integral technique for the crack analysis becomes a promising computational tool. Furthermore, to provide an efficient and accurate computational algorithm, an analytical solution concerning only the crack boundary \( \Gamma^*_e \) of infinite bodies is embedded into the boundary element formulation. To find the analytical closed form solution for an infinite homogeneous piezoelectric material containing a crack subject to arbitrary loadings, an extended version of Stroh formalism [12] considering the constitutive law of an anisotropic linear piezoelectric medium, the kinematic relation of small deformation and the static equilibrium equations [9] has been employed.

A computational algorithm based upon the above concept has been developed recently for the analysis of the defects in piezoelectric materials [13]. To understand whether the stress intensity factors are qualified to be detectors for crack identification, we now study the sensitivity of stress intensity factors on the crack size and location by using the numerical method described above.

**Examples**

Consider a uniformly loaded cracked plate as shown in Fig. 1. The applied tensile stress \( \sigma = 1\, \text{Pa} \); the plate width \( W \) and length \( L \) are \( W = 10\, \text{cm}, L = 30\, \text{cm} \); the material of the plate is PZT4 whose properties are \( S_{11} = 10.9 \times 10^{-12} \, \text{m}^2/\text{Nt}, S_{12}/S_{11} = -0.497, S_{13}/S_{11} = -0.193, S_{33}/S_{11} = 0.725, S_{44}/S_{11} = 1.771, S_{66}/S_{11} = 2.994; g_{33} = 2.61 \times 10^{-2} \, \text{Volt.m/Nt}, g_{13}/g_{33} = -0.425, g_{15}/g_{33} = 1.51; \epsilon_{11} = 8.697 \times 10^7 \, \text{Volt}^2/\text{Nt}, \epsilon_{33}/\epsilon_{11} = 0.881, \) where \( S_{ij}, g_{ij} \) and \( \epsilon_{ij} \) denote, respectively, elastic compliances, piezoelectric constants and permittivities. The variation of the crack is represented by its center location \( (d_1, d_2) \) and length \( a \) with unit cm. From Fig. 1 we see that the larger the crack length \( a \) the higher the stress intensity factor \( K_I \). The variation of the crack center location \( (d_1, d_2) \) also influences the stress intensity factors. However, when the crack size is under a certain value such as \( a = 1.0 \, \text{cm} \) in Fig. 1, it is very difficult to distinguish the difference from different location with the same small crack size. Moreover, the variation of \( d_2 \) has almost no influence on the value of the stress intensity factor. Therefore, we may conclude that the stress intensity factors are qualified to be detectors for the crack size identification. Under certain restrictions, it may also be applied to detect the crack location.
CRACK IDENTIFICATION

Consider that the internal crack boundary $\Gamma_c^*$ is not known. A set of experimental measurements are available as

$$u_k(\mathcal{X}_m, \Gamma_c^*) = \mathcal{Z}_k(\mathcal{X}_m, \Gamma_c^*), \quad \mathcal{X}_m \in \Gamma_{eu},$$

$$t_k(\mathcal{X}_n, \Gamma_c^*) = \mathcal{T}_k(\mathcal{X}_n, \Gamma_c^*), \quad \mathcal{X}_n \in \Gamma_{et},$$

$m = 1, 2, ..., M; n = 1, 2, ..., N$. A total of $(N+M)$ experimental observations are thus available. The identification problem consists of determining the crack geometry $\Gamma_c^*$ based upon these given experimental values.

In section 2, the crack analysis shows that the stress intensity factor is really a promising detector for the crack size identification. However, it is not sensitive enough for the crack location. Therefore, in this section only the crack size will be identified by using the stress intensity factors as our detectors. Thus, the design variable $\Gamma_c^*$ can be represented by the crack size $a$. Moreover, the experimental values $u_k(\mathcal{X}_m, \Gamma_c^*)$ and $t_k(\mathcal{X}_n, \Gamma_c^*)$ should be utilized to calculate the measured stress intensity factors $K_i(a^*)$. The identification problem now becomes the determination of the crack size $a^*$ based upon $K_i(a^*)$. To proceed with this kind of inverse problem, the techniques of nonlinear optimization [14] and neural network [15] are applied in this paper.

(i) NONLINEAR OPTIMIZATION

To determine the crack geometry $a^*$ satisfying the measured stress intensity factors $K_i(a^*)$, a crack size $a$ is assumed. The variation of the stress intensity factors in the assumed internal crack configuration with respect to the actual internal crack configuration is defined as

$$\Delta K_i(a) = K_i(a^*) - K_i(a).$$

The objective is to fit the numerical results $K_i(a)$, obtained by the boundary element method described in section 2, to the measured $K_i(a^*)$ by varying $a$. The best fit is obtained by minimizing the objective function $F(a)$, which is the sum of the square of the residuals $\Delta K_i$, with respect to $a$. That is,

$$\text{minimize:} \quad F(a) = \sum_{i=1}^{4} \left[ \frac{K_i(a)}{K_i(a^*)} - 1 \right]^2,$$

in which squares are taken to avoid cancellation between the residuals $\Delta K_i$ of opposite sign. It is desired that the difference between $K_i(a)$ and $K_i^*$ is within a small tolerance $\epsilon$, and the crack is inside the bodies. The constraints are then written as follows:

$$g_i = \left| \frac{K_i(a)}{K_i(a^*)} - 1 \right| - \epsilon < 0, \quad i = 1, 2, 3, 4,$$

$$0.02W < a < 0.98W,$$

where Eqn(6b) represents side constraints on the crack length to prevent the optimizer from proposing a meaningless design, and $W$ is related to the size of the bodies considered. Since both the objective function $F$ and constraints $g_i$ are implicit nonlinear functions of the design variable $a$, the problem listed in Eqns(5,6) belongs to the nonlinear constrained minimization problems.
In general, there are two strategies to solve nonlinear optimization problems. One is sequential unconstrained minimization technique, e.g., the penalty function method and the augmented Lagrange multiplier method, etc., the other is direct method, e.g., the sequential linear programming and sequential quadratic programming, etc. To proceed with these two strategies, we need optimizers such as Davidon-Fletcher-Powell (DFP) variable metric method for unconstrained minimization and the method of feasible directions for constrained minimization to find the search directions. After finding the search direction, an updated design variable can be determined by the techniques of one-dimensional search such as the polynomial approximation and the golden section method. In other words, the optimization is an iterative procedure given by

\[ \tilde{X}^q = \tilde{X}^{q-1} + \alpha^* \tilde{S}^q \]

where \( q \) is the iteration number and \( \tilde{S} \) is a vector search direction in the design space. Beginning from the initial design variable \( \tilde{X}^0 \), the design is updated iteratively until all the convergence criteria have been satisfied. [14]

With the understanding of the optimization procedure, two different combinations are used in this paper: (1) augmented Lagrange multiplier method + DFP variable metric method + polynomial extrapolation, (2) the method of feasible directions + golden section method followed by polynomial interpolation. The public domain program COPES/ADS [20] is chosen to implement these two combinations.

**Examples**

As stated previously, the attention of this paper will be focused on the identification of crack size not its location due to the sensitivity analysis of Section 2. Although it looks trivial without identifying the crack location, the results can reveal the potential of some identification techniques which may be helpful for the future study. Since only the size of the crack is concerned, for simplicity the examples shown below will be a piezoelectric material containing an unknown single centered crack. The goal is now becoming that identify the crack size \( a^* \) by the given experimental data \( u_k^x \) and \( t_k^x \) measured along the plate boundary. Before implementing the optimization software, the measured \( u_k^x \) and \( t_k^x \) should be transformed to the measured stress intensity factors \( K_i(a^*) \) through any analysis means such as J-integral or the definition given in Eqn(1) in which the interior stresses may be calculated from the results of boundary element programming. Although this is important to the entire identification process, it is still in the developing status. In this paper, we just assume that \( K_i(a^*) \) is known.

Consider the same geometry (except now the centered crack size is unknown), loading and material conditions as shown in Fig. 1. If \( K_i(a^*) \) is known to be \( K_{I} = 0.1305 Pa \sqrt{cm} \), \( K_{II} = K_{III} = K_{IV} = 0 \), the crack size is now identified as \( a^* = 1.001 cm \) through the use of combination (1) after 14 iteration. With combination (2), the crack size is identified as \( a^* = 1.004 cm \) after 24 iteration. By comparing with Fig. 1, we see that the crack size we identify is correct.

**(ii) NEURAL NETWORK**

An artificial neural network is a parallel, distributed information processing structure consisting of processing elements interconnected with weights. In this paper, a most popular learning scheme called the back-propagation neural network (BPN) will be applied for implementing the crack size identification. A typical architecture of BPN contains three basic layers - input, output and hidden layers. According to Kolmogorov's mapping theorem [21], any continuous function
could be mapped exactly by a three-layer feedforward neural networks. Mathematically, the relation between these three layers may be written as

\[ H_k = f(\text{net}_k) = \sum_i V_{ik}P_i - h_k, \]
\[ O_j = f(\text{net}_j) = \sum_k W_{kj}H_k - o_j, \]  

where \( P_i, O_j \) and \( H_k \) denote, respectively, the values of input, output and hidden neurons; \( V_{ik} \) and \( W_{kj} \) are the weights of input-hidden and hidden-output layers, respectively; \( h_k \) and \( o_j \) are the hidden and output thresholds, respectively; \( f(\cdot) \) is the transfer function. By Eqn(7), the input pattern \( P_i \) is propagated forward, and the calculated responses \( O_j \) are obtained. The errors between the desired outputs \( d_j \) and the calculated outputs \( O_j \) then propagate backward through the network, providing vital information for weight adaptation. By the generalized Delta learning rule [15] with the errors defined as

\[ Error = \frac{1}{2} \sum_j (O_j - d_j)^2, \]

the weights should be updated by adding

\[ \Delta V_{ik}(n) = \eta \delta_k P_i + \alpha [V_{ik}(n) - V_{ik}(n - 1)], \]
\[ \Delta W_{kj}(n) = \eta \delta_j H_k + \alpha [W_{kj}(n) - W_{kj}(n - 1)]. \]  

where

\[ \delta_k = f'(\text{net}_k) \sum_j (\delta_j W_{jk}), \]
\[ \delta_j = (d_j - O_j)f'(\text{net}_j). \]

\( \eta \) and \( \alpha \) are, respectively, the learning rate and momentum coefficients which govern the step size of the training process. The thresholds are updated by adding \( \Delta o_j = -\eta \delta_j, \Delta h_k = -\eta \delta_k. \)

**Examples**

For the case of crack identification studied in this paper, the training patterns \( (P_j, d_j)_k \) are \( (K_I, K_{II}, K_{III}, K_{IV}, a)_k \) provided by the boundary element program stated in Section 2. Consider the same problem as that discussed in nonlinear optimization. The results of identification by the method of BPN are presented in Fig. 2. From this figure we see that the more the training patterns the better the identification, and this tendency will not be affected by the learning rate \( \eta \), momentum coefficient \( \alpha \) and the hidden neurons' number.

**DISCUSSION AND CONCLUDING REMARKS**

The purpose of this paper is to explore the feasibility of identifying a crack by using only the data from static deformation. To this end, a very special case for a piezoelectric material containing an unknown single centered crack has been studied. The crack analysis (forward problem) is performed by combining the Stroh formalism with the boundary element method. The crack identification (inverse problem) is then implemented by applying the techniques of nonlinear optimization and artificial neural network. Although it looks trivial without identifying the crack location, the present results may be helpful for our future study of crack identification in more realistic engineering problems. Thus, the next step challenging us is to find an appropriate
parameter which is sensitive to the crack location, and can be used to develop a new technique for crack identification.

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REFERENCES


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**Figure 1** Variation of the right tip stress intensity factor $K_r$ with respect to the crack size $a$ and location $(d_1, d_2)$

**Figure 2** Crack size identification by BPN. $\eta$: learning rate, $\alpha$: momentum coefficient, $N_h$: number of hidden layer nodes.