Stroh Formalism for Thermoelastic Problems

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Overview

In the literature, there are two different complex variable formulations for two-dimensional linear anisotropic elasticity. One is the Lekhnitskii formalism [1, 2] which starts with the equilibrated stress functions followed by compatibility equations, and the other is the Stroh formalism which starts with the compatible displacements followed by equilibrium equations. The improvement and maturity of Stroh formalism relies on the development of eigen-relations and identities among the elasticity constants, which makes the mathematical formulation and manipulation simpler and easier. Otherwise, a complicated form solution will be obtained or an intractable mathematical problem will be left. The discussions of the structures, identities, and related transformation of the elasticity matrices involved in the Stroh formalism as well as its applications to elasticity problems have been well documented in the books of Ting [3] and Hwu [4]. The extended version dealing with thermoelasticity can also be found in these two books. In this entry, the extended Stroh formalism for two-dimensional anisotropic thermoelasticity described in [4] will be briefly stated. Its applications to the anisotropic bodies containing cracks, holes, inclusions, or interfaces will also be shown and discussed based upon the results presented in [4].

Composite laminates are increasingly being used not only in traditional areas like aerospace but also in many engineering applications. Some of these applications are the structures under thermal environment. The asymmetry of laminates will cause coupling between stretching and bending, which may complicate the analysis. Due to the desirable characteristics of composite laminates, sometimes the engineering designers want to utilize the coupling effects to do something that cannot be achieved by using metallic or symmetric laminates. Thus, the study of thermal stresses in unsymmetric laminates becomes important for practical engineering design. Like the extension of Stroh formalism to anisotropic thermoelasticity, in this entry we will also present the extended Stroh-like formalism for thermal stress analysis of general laminates, whose detailed formulation and applications can also be found in [4].

Stroh Formalism for Two-Dimensional Thermoelasticity

In a fixed rectangular coordinate system $x_i, i = 1, 2, 3$, let $u_i, \sigma_{ij}, e_{ij}, T$ and $h_i$ be, respectively, the displacement, stress, strain, temperature, and heat flux. The equations of motion and the equilibrium equations are

$$\nabla^4 u_i = \rho \ddot{u}_i,$$

$$\nabla \cdot \sigma_{ij} = 0,$$

where $\rho$ is the density, $\nabla$ is the gradient operator, and $\ddot{u}_i$ is the acceleration of the displacement $u_i$. The constitutive equations relate the stress tensor $\sigma_{ij}$ to the strain tensor $e_{ij}$ and the temperature $T$ as

$$\sigma_{ij} = C_{ijkl} e_{kl} + \lambda T \delta_{ij},$$

where $C_{ijkl}$ is the elastic stiffness tensor, $\lambda$ is the thermal expansion coefficient, and $\delta_{ij}$ is the Kronecker delta. The thermal stress can be expressed as

$$\sigma_{ij}^T = \alpha T \delta_{ij}$$

where $\alpha$ is the coefficient of thermal expansion. The total stress tensor $\sigma_{ij}$ is then

$$\sigma_{ij} = C_{ijkl} e_{kl} + \alpha T \delta_{ij}.$$