OUTLINE FOR Chapter 4

AIRFOIL NOMENCLATURE

The leading edge circle: (usually radius = 0.02 chord length c)
The trailing edge:
The chord line: Straight line connecting the center of leading edge circle and the trailing edges.
The leading edge:
The chord length c: the length between the leading edge and the trailing edge.
The thickness t: the distance between upper and lower surfaces, measured normal to the chord line.
The mean camber line: the locus of points halfway between the upper and lower surfaces, measured normal to the chord line.
camber: the maximum distance between the mean camber line and the chord line.
The distances of the points of maximum thickness and maximum camber aft of the leading edge.
NACA AIRFOIL NOMENCLATURE

NACA “four-digit” series: (http://airfoiltools.com/airfoil/naca4digit)
Example: NACA 2412 (2% camber at 40% chord, with 12% thickness)
  2 => maximum camber = 0.02c
  4 => maximum camber located at 0.4c from the leading edge.
  12 => maximum thickness = 0.12c.

NACA “five-digit” series: (http://airfoiltools.com/airfoil/naca5digit)
Example: NACA 23012
  2 => Maximum camber = 0.02c and 2*3/2=3 => lift coefficient C_l = 0.3
  30 => 30/2=15 => Maximum camber = 0.02c located at 0.15c from the leading edge.
  12 => Maximum thickness = 0.12c.

NACA 6-series:
Example: NACA 65-218
  6 => series designation
  5 => the minimum pressure occurs at 0.5c for the basic symmetric thickness distribution at zero lift
  2 => the design lift coefficient 0.2
  18 => Maximum thickness =0.18c

Interactive NACA Airfoil Shape

http://airfoiltools.com/airfoil/naca4digit
NACA “Four-Digit” Series:

NACA0000

NACA0012

NACA0012

NACA0000

NACA4400

NACA4402

NACA4412

NACA4412

NACA4424

VARIATION OF PRESSURE DISTRIBUTION OVER AN AIRFOIL WITH ANGLE OF ATTACK

Inviscid flow $C_f=0$

\[
\begin{align*}
c_a &= \frac{1}{c} \int_0^l (C_{pa} - C_{p,\infty}) \, dx \\
c_d &= \frac{1}{c} \int_{LE}^{TE} (C_{p,\infty} - C_{p,\infty}) \, dy \\
c_m &= \frac{1}{c} \left[ \int_0^l (C_{p,\infty} - C_{p,\infty}) \, dx + \int_{LE}^{TE} (C_{p,\infty} - C_{p,\infty}) \, dy \right] \\
c_t &= c_a \cos \alpha - c_d \sin \alpha \\
c_d &= c_a \sin \alpha + c_d \cos \alpha
\end{align*}
\]
THE LIFT CURVE

Lift decrease, Drag increase

Max lift coefficient $c_{l, \text{max}}$

Lift due to flow separation

$C_l = a_0 (\alpha - \alpha_{L=0})$

$a_0 = 2 \pi$

Lift Coefficient for NACA0012

$L_{\text{max}} = 1.288$

Lift slope ~ $2 \pi$

Stall angle ~ $14^\circ$

Zero-lift angle of attack $\alpha_{L=0} = 0$

Angle of attack $\alpha$
DARG FOR SUBSONIC 2-D AIRFOIL

$D_f$: skin friction drag, $C_f = D_f / (1/2 \rho V_\infty^2 S)$

$D_p$: pressure drag ($D_f \gg D_p$ in small angle of attack)

Profile drag coefficient $C_d = (D_f + D_p) / (1/2 \rho V_\infty^2 S)$

Drag Coefficient $C_d$ for NACA0012

Angle of Attack ($\alpha$) vs Drag Coefficient $C_d$
Moment Coefficient $C_{m,le}$ and $C_{m,c/4}$ for NACA0012

For low speed airfoil, aerodynamic center is always at or near the quarter-chord point, i.e. c/4 from leading edge.
OUTLINE FOR Chapter 4

THEORETICAL SOLUTION FOR LOW SPEED FLOW OVER AIRFOIL (I) - THE PHILOSOPHY

Thickness effect
Angle of attack effect
Camber effect
Thin airfoil theory
THE VORTEX SHEET

The Vortex Sheet

Define $\gamma = \gamma(s)$ as the strength of the vortex sheet, per unit length along $s$.

$$\Gamma = \int \gamma ds$$

$$d\phi = \frac{-\gamma ds}{2\pi}$$

$$\Gamma = - (u_0 \, dn - u_1 \, ds - v_0 \, dn + u_2 \, ds)$$

$$\Gamma = (u_1 - u_2) \, ds + (v_1 - v_2) \, dn$$

$$\gamma \, ds = (u_1 - u_2) \, ds + (v_1 - v_2) \, dn$$

let $dn \to 0$

$$\gamma = u_1 - u_2$$

the local jump in tangential velocity across the vortex sheet is equal to the local sheet strength.

THE KUTTA CONDITION (I)

Kutta-Joukowsk theorem:

$$L' = \rho V \Gamma$$

Kutta condition:

For a given airfoil at a given angle of attack, the value of $\Gamma$ around the airfoil is such that the flow leaves the trailing edge smoothly.
THE KUTTA CONDITION (II)

Finite trailing edge angle  
Finite angle

Cusp trailing edge angle  
Cusp

At point $a$: $V_1 = V_2 = 0$  
At point $a$: $V_1 = V_2 \neq 0$

$p_s + \frac{1}{2} \rho V_1^2 = p_s + \frac{1}{2} \rho V_2^2$

$V_1 = V_2$

Kutta condition expressed in terms of the strength of the vortex sheet is

$\gamma(\text{TE}) = \gamma(a) = V_1 - V_2$

$\gamma(\text{TE}) = 0$
Application in 2D Airfoil Lift Calculation

- The following equations are unconditional satisfied (no assumptions).

\[ \int \nabla \cdot \vec{V} \, dV = \int \vec{V} \cdot n \, dS = 0 \]

\[ \Gamma = \oint \nabla \times \vec{V} \cdot n \, dA = \int \omega \eta \, dA \]

Upstream
Uniform velocity
\[ U_\infty = V \]

Near airfoil
Irrotational flow
\[ \omega = 0 \]

Far downstream
Uniform velocity
\[ U_\infty = V \]

Irrotational flow
\[ \omega = 0 \]

Application in 2D Airfoil Lift Calculation

- Downwash velocity \( W \)

\[ \Gamma = \frac{L}{\rho V b w h} \]

\[ L = \rho V b w h \]

\[ D = \rho w b \Gamma \]

OUTLINE FOR Chapter 4
CLASSICAL THIN AIRFOIL THEORY (I)

Calculate $\gamma(s)$ such that the camber line becomes a streamline of the flow and such that the Kutta condition is satisfied at trailing edge, i.e., $\gamma(TE) = 0$

Kutta-Joukowski theorem:
$$\Gamma = \int \gamma ds$$
$$L' = \rho a V_s \Gamma$$

THE THIN AIRFOIL THEORY (II)

Calculate $\gamma(x)$ such that the camber line becomes a streamline of the flow and such that the Kutta condition is satisfied at trailing edge, i.e., $\gamma(c) = 0$. 

$V_{x,A} + w'(s) = 0$
$$V_{w,A} = V_\infty \sin \left[ \alpha + \tan^{-1} \left( \frac{dz}{dx} \right) \right]$$
$$\sin \theta \approx \tan \theta \approx \theta \text{ for small } \theta,$$
$$V_{w,A} = V_\infty \left( \alpha - \frac{dz}{dx} \right)$$
THE THIN AIRFOIL THEORY (III)

Solve integral equation $\gamma(\xi)$ to satisfy (1) the camber line is a streamline and (2) Kutta condition $\gamma(c) = 0$ boundary conditions.

**Fundamental equation for thin airfoil theory**

Solve integral equation $\gamma(\xi)$ to satisfy (1) the camber line is a streamline and (2) Kutta condition $\gamma(c) = 0$ boundary conditions.

THE SYMMETRIC AIRFOIL - A FLAT PLATE WITH ANGLE OF ATTACK

- No camber, camber line = chord line $dz/dx = 0$

\[
\frac{1}{2\pi} \int_{0}^{c} \frac{\gamma(\xi)}{x - \xi} d\xi = V_a (\alpha - \frac{dz}{dx})
\]

Mathematical theory of integral equations which satisfy Kutta condition $\gamma(\pi) = 0$ by using L’Hospital rule.
LIFT AND LIFT COEFFICIENT OF FLOW OVER A FLAT PLATE WITH ANGLE OF ATTACK $\alpha$

\[ \Gamma = \int_{0}^{\pi} \gamma(\xi) \, d\xi \]
\[ \xi = \frac{c}{2} (1 - \cos \theta) \]
\[ d\xi = \frac{c}{2} \sin \theta \, d\theta \]
\[ \Gamma = \frac{c}{2} \int_{0}^{\pi} \gamma(\theta) \sin \theta \, d\theta \]

\[ \gamma(\theta) = 2\alpha V_\infty \left(1 + \cos \theta \right) \frac{1}{\sin \theta} \]

Kutta-Joukowski theorem:

\[ \Gamma = \alpha c V_\infty \int_{0}^{\pi} (1 + \cos \theta) \, d\theta = \pi \alpha c V_\infty \]

\[ L' = \rho_\infty V_\infty \Gamma = \pi \alpha c \rho_\infty c V_\infty^2 \]

\[ c_l = \frac{L'}{q_\infty S} \quad S = c(1) \]

\[ c_l = 2\pi \alpha \quad \text{Lift slope} = \frac{dc_l}{d\alpha} = 2\pi \]

\[ \frac{\pi \alpha}{2} \]

\[ L' = \rho_\infty V_\infty \Gamma \]

\[ \frac{\pi V_\infty^2}{4} \]

\[ \frac{\pi V_\infty^2}{4} \]

\[ \frac{\pi V_\infty^2}{4} \]

Moments and Moment Coefficient of Flow Over a Flat Plate with Angle of Attack

\[ dL = \rho V_\infty d\Gamma = \rho V_\infty \gamma(\xi) d\xi \]

\[ d\Gamma = \gamma(\xi) \, d\xi \]

\[ \xi = \frac{c}{2} (1 - \cos \theta) \]

\[ d\xi = \frac{c}{2} \sin \theta \, d\theta \]

\[ \frac{\pi}{2} \]

\[ \frac{\pi}{2} \]

\[ c_{m,le} = \frac{M'_{LE}}{q_\infty c^2} \quad \text{where} \quad S = c(1) \]

\[ \pi \alpha = \frac{c_l}{2} \quad \rightarrow \quad c_{m,le} = -\frac{c_l}{4} \]

\[ \pi \alpha = \frac{c_l}{2} \quad \rightarrow \quad c_{m,le} = -\frac{c_l}{4} \]

\[ c_{m,le} = c_{m,le} + \frac{c_l}{4} \quad \rightarrow \quad c_{m,le/4} = 0 \]

For a thin, symmetric airfoil, the aerodynamic center is located at the $c/4$ location.
EXPERIMENTAL V.S. THEORETICAL $C_l$ AND $C_{m,c/4}$ OF NACA0012 SYMMETRIC AIRFOIL

Conclusion:
1. $C_l = 2\pi \alpha$.
2. Lift slope = $2\pi$.
3. The center of pressure and the aerodynamic center are both located at the quarter-chord point.

OUTLINE FOR LECTURE 10

- Incompressible flow over airfoil
  - Airfoil aerodynamics
    - Experimental data
      - A critical approach to specifying airfoil shapes
  - Airfoil characteristics
    - Singularity distribution along the airfoil surface
      - Method of integration
        - Singularity distribution along the airfoil surface
          - Karman's theorem and the starting vortex
            - Theory of arbitrary airfoil shapes
              - NACA aerodynamic theory
                - Stress analysis of an airfoil
THE CAMBERED AIRFOIL WITH ANGLE OF ATTACK (I)

\[ \frac{1}{2\pi} \int_0^\pi \gamma(\xi) d\xi = V_0 \left( \frac{\alpha - dz}{dx} \right) \]

\[ \frac{1}{2\pi} \int_0^\pi \gamma(\theta) \sin \theta d\theta = V_0 \left( \frac{\alpha - dz}{dx} \right) \]

\[ \gamma(\theta) = 2V_0 \left( \sum A_n \frac{1 + \cos \theta}{\sin \theta} \right) \]

To find \(A_0\) and \(A_n\) (\(n = 1, 2, 3, \ldots\)) in order that the camber line be a streamline of the flow.

\[ A_0 = \sum_{n=1}^{\infty} A_n \cos \theta \phi = \alpha - \frac{dz}{dx} \]

\[ \frac{dz}{dx} = (\alpha - A_0) + \sum_{n=1}^{\infty} A_n \cos \theta \phi \]

Fourier cosine series expansion of the function \(dz/dx\).

THE CAMBERED AIRFOIL WITH ANGLE OF ATTACK (II)

Fourier cosine series expansion of a function \(f(\theta)\) over an interval \(0 \leq \theta \leq \pi\)

\[ f(\theta) = B_0 + \sum_{n=1}^{\infty} B_n \cos n\theta \]

\[ B_0 = \frac{1}{\pi} \int_0^\pi f(\theta) d\theta \]

\[ B_n = \frac{2}{\pi} \int_0^\pi f(\theta) \cos n\theta d\theta \]

\[ \frac{dz}{dx} = (\alpha - A_0) + \sum_{n=1}^{\infty} A_n \cos n\theta \phi \]

\[ A_0 = \alpha - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta \]

\[ A_0 \text{ depends on both } \frac{dz}{dx} \text{ and } \alpha \]

\[ A_n = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos n\theta_0 d\theta_0 \]

\(A_0\) depends on the shape of the camber line, \(dz/dx\).

AERODYNAMICS (W4-I-2)

AERODYNAMICS (W4-I-3)
LIFT AND LIFT COEFFICIENT OF A CAMBERED AIRFOIL

\[
\Gamma = \int_0^c \gamma(\xi) \, d\xi \\
\xi = \frac{c}{2} (1 - \cos \theta) \\
d\xi = \frac{c}{2} \sin \theta \, d\theta
\]

\[
\Gamma = \frac{c}{2} \int_0^\pi \gamma(\theta) \sin \theta \, d\theta
\]

\[
\Gamma = cV_a \left[ A_0 \int_0^{\pi/2} (1 + \cos \theta) \, d\theta + \sum_{n=1}^{\infty} A_n \int_0^{\pi/2} \sin n\theta \sin \theta \, d\theta \right]
\]

\[
\Gamma = cV_a \left( \pi A_0 + \frac{\pi}{2} A_1 \right)
\]

\[
A_0 = \alpha - \frac{1}{\pi} c_1 \int_0^\pi \cos n\theta \, d\theta
\]

\[
A_1 = \frac{2}{\pi} \int_0^\pi \cos n\theta \, d\theta
\]

Lift slope: \( \frac{d\gamma}{d\alpha} = 2\pi \)

\[ c_l = 2\pi (\alpha - \alpha_{\infty}) \]

\[ \alpha_{\infty} = -\frac{1}{\pi} \int_0^\pi \frac{dz}{dx} (\cos \theta_i - 1) \, d\theta_i \]

\[ \alpha_{\infty} = 0 \] for a symmetric airfoil

Zero-Lift Angle of Attack

\[ \alpha_{\infty} = 0 \] for a symmetric airfoil

\[ \alpha_{\infty} = \frac{\pi}{2} \] for an n = 1

\[ \alpha_{\infty} = 0 \] for an \( n \neq 1 \)

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AERODYNAMICS (W4-1-4)

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AERODYNAMICS (W3-1-5.1)

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Figure 5.6 Comparison of lift curves for cambered and symmetric airfoils.
**MOMENT AND MOMENT COEFFICIENT OF A CAMBERED AIRFOIL**

\[ M_{LE} = \int_0^\infty \xi dL = -\rho V_c \int_0^\infty \xi \gamma \xi d\xi \]

\[ \gamma(\theta) = 2V_c \left( \frac{1 + \cos \theta}{\sin \theta} + \sum \frac{A_n \sin n\theta}{n} \right) \]

\[ M_{LE} = -D \rho \frac{V_c^2}{2} \frac{S}{c} \]

\[ c_{u,le} = \frac{M_{LE}}{q_S c} \quad \text{where} \quad S = c(1) \]

\[ c_{u,k} = -\frac{\pi}{2} \left( A_0 + A_1 - \frac{A_2}{2} \right) \]

\[ c_1 = \frac{1}{2} \rho V_c^2 c(1) = \pi (2A_0 + A_1) \]

\[ x_{cp} = -\frac{M_{LE}}{L'} = \frac{c_{u,le} c}{c_1} \]

**SUMMARY**

<table>
<thead>
<tr>
<th>Symmetric Airfoil</th>
<th>Cambered Airfoil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chordwise circulation distribution, ( \gamma )</td>
<td>( 2\pi \left[ A_0 + \cos \theta + \sum \frac{\sin n\theta}{n} \right] )</td>
</tr>
<tr>
<td>Lift coefficient, ( c_L )</td>
<td>( 2\pi \left[ A_0 + \frac{1 + \cos \theta}{\sin \theta} + \sum \frac{A_n \sin n\theta}{n} \right] )</td>
</tr>
<tr>
<td>Slope of ( c_L ) vs. ( \alpha ) curve, ( \beta_0 )</td>
<td>( 2\pi )</td>
</tr>
<tr>
<td>Chordwise location of center of pressure, ( x_{cp} )</td>
<td>( \frac{\pi}{4} )</td>
</tr>
<tr>
<td>Moment coefficient about leading edge, ( c_{m,le} )</td>
<td>( -\frac{\pi}{2} )</td>
</tr>
<tr>
<td>Aerodynamic center</td>
<td>( x_{ac} = \frac{\pi}{4} (A_2 - A_0) )</td>
</tr>
<tr>
<td>Moment coefficient about aerodynamic center, ( c_{m,ac} )</td>
<td>( -\frac{\pi}{4} (A_2 - A_0) )</td>
</tr>
<tr>
<td>Angle of zero lift, ( \alpha_{0} )</td>
<td>( 2\pi )</td>
</tr>
</tbody>
</table>
Example 4.2. Consider an NACA 23012 airfoil. The mean camber line for this airfoil is given by
\[ \frac{x}{c} = 2.6595 \left[ \frac{x}{c} \right]^3 - 0.6075 \left( \frac{x}{c} \right)^2 + 0.1147 \left( \frac{x}{c} \right) \]
for \( 0 \leq \frac{x}{c} \leq 0.2025 \)
and \[ \frac{x}{c} = 0.02208 \left( 1 - \frac{x}{c} \right) \]
for \( 0.2025 \leq \frac{x}{c} \leq 1.0 \).

Calculate (a) the angle of attack at zero lift, (b) the lift coefficient when \( \alpha = 4^\circ \), (c) the moment coefficient about the quarter chord, and (d) the location of the center of pressure in terms of \( x_{cp}/c \), when \( \alpha = 4^\circ \). Compare the results with experimental data.

Solution:
\[ \frac{dz}{dx} = 2.6595 \left[ 3 \left( \frac{x}{c} \right)^2 - 1.215 \left( \frac{x}{c} \right) + 0.1147 \right] \]
for \( 0 \leq \frac{x}{c} \leq 0.2025 \)

\[ = -0.02208 \]
for \( 0.2025 \leq \frac{x}{c} \leq 1.0 \)

\[ x = \left( \frac{c}{2} \right) (1 - \cos \theta) \]
\[ \frac{dz}{dx} = 0.6840 - 2.3736 \cos \theta + 1.995 \cos^2 \theta \]
for \( 0 \leq \theta \leq 0.9335 \) rad

\[ = -0.02208 \]
for \( 0.9335 \leq \theta \leq \pi \)

Mathematica 8 Tutor: http://www.youtube.com/watch?v=MjCpgxYslRc
\[
A_0 = \alpha - 0.0286 \quad A_1 = 0.0954 \quad A_2 = 0.0792
\]
\[
c_l = \pi (2A_0 + A_1) \quad c_{m_{\alpha}} = \frac{\pi}{4} (A_2 - A_1) \quad X_{cp} = \frac{c}{4} \left[ 1 + \frac{\pi}{C_l} (A_1 - A_2) \right]
\]
(a) zero-lift angle of attack:
\[
c_l = \pi (2A_0 + A_1) = \pi [2(\alpha - 0.0286) + 0.0954] = 0
\]
\[
\alpha_{L=0} = -0.0191 \text{ rad} = -1.09^\circ
\]
(b) lift coefficient at $\alpha=4^\circ$:
\[
c_l = \pi (2A_0 + A_1) = \pi \left[ 2 \left( \frac{4\pi}{180} - 0.0286 \right) + 0.0954 \right] = 0.559
\]
(c) moment coefficient about the quarter chord:
\[
c_{m_{\alpha}} = \frac{\pi}{4} (A_2 - A_1) = \frac{\pi}{4} (0.0792 - 0.0954) = -0.0127
\]
(d) center of pressure at $\alpha=4^\circ$:
\[
\frac{X_{cp}}{c} = \frac{1}{4} \left[ 1 + \frac{\pi}{0.559} (0.0954 - 0.0792) \right] = 0.273
\]